

Critical Points

Find every critical coordinate, classify it, and give the maximal open intervals where the polynomial increases and decreases.

1. Find and classify all critical points; give increasing/decreasing intervals.

f(x) = x² + 4 · x + 3

2. Find and classify all critical points; give increasing/decreasing intervals.

f(x) = x³ - 6 · x² + 9 · x - 1

3. Find and classify all critical points; give increasing/decreasing intervals.

f(x) = - x⁴ - 8 · x³ - 18 · x² - 8 · x + 12

4. Find and classify all critical points; give increasing/decreasing intervals.

f(x) = x² - 1

Critical Points

Find every critical coordinate, classify it, and give the maximal open intervals where the polynomial increases and decreases.

5. Find and classify all critical points; give increasing/decreasing intervals.

$f(x) = -2 \cdot x^3 + 4 \cdot x - 2$

6. Find and classify all critical points; give increasing/decreasing intervals.

$f(x) = 2 \cdot x^4 - 8 \cdot x^3 + 4 \cdot x^2 + 8 \cdot x - 10$

7. Find and classify all critical points; give increasing/decreasing intervals.

$f(x) = x^2 + 4 \cdot x + 7$

8. Find and classify all critical points; give increasing/decreasing intervals.

$f(x) = -x^3 + 6 \cdot x^2 - 12 \cdot x + 7$

Critical Points
Worked Answer Key

Find every critical coordinate, classify it, and give the maximal open intervals where the polynomial increases and decreases.

1. Differentiate and analyze signs.

$f(x) = x^2 + 4 \cdot x + 3$
 $f'(x) = 2 \cdot x + 4$
Zeros: -2 (m1)
f' signs: (-inf, -2) - (-2 , +inf) +
(-2 , -1) local minimum
Increasing: (-2 , +inf)
Decreasing: (-inf, -2)

2. Differentiate and analyze signs.

$f(x) = x^3 - 6 \cdot x^2 + 9 \cdot x - 1$
 $f'(x) = 3 \cdot x^2 - 12 \cdot x + 9$
Zeros: 1 (m1) 3 (m1)
f' signs: (-inf, 1) + (1, 3) - (3, +inf) +
(1, 3) local maximum
(3, -1) local minimum
Increasing: (-inf, 1) (3, +inf)
Decreasing: (1, 3)

3. Differentiate and analyze signs.

$f(x) = -x^4 - 8 \cdot x^3 - 18 \cdot x^2 - 8 \cdot x + 12$
 $f'(x) = -4 \cdot x^3 - 24 \cdot x^2 - 36 \cdot x - 8$
Zeros: - $\sqrt{3} - 2$ (m1) -2 (m1) $\sqrt{3} - 2$ (m1)
f' signs: (-inf, - $\sqrt{3} - 2$) + (- $\sqrt{3} - 2$, -2) - (-2 , $\sqrt{3} - 2$) + ($\sqrt{3} - 2$, +inf) -
(- $\sqrt{3} - 2$, 13) local maximum
(-2 , 4) local minimum
($\sqrt{3} - 2$, 13) local maximum
Increasing: (-inf, - $\sqrt{3} - 2$) (-2 , $\sqrt{3} - 2$)
Decreasing: (- $\sqrt{3} - 2$, -2) ($\sqrt{3} - 2$, +inf)

4. Differentiate and analyze signs.

$f(x) = x^2 - 1$
 $f'(x) = 2 \cdot x$
Zeros: 0 (m1)
f' signs: (-inf, 0) - (0, +inf) +
(0, -1) local minimum
Increasing: (0, +inf)
Decreasing: (-inf, 0)

Critical Points
Worked Answer Key

Find every critical coordinate, classify it, and give the maximal open intervals where the polynomial increases and decreases.

5. Differentiate and analyze signs.

$f(x) = -2 \cdot x^3 + 4 \cdot x - 2$
 $f'(x) = -6 \cdot x^2 + 4$
Zeros: $\frac{-1}{3} \cdot \sqrt{6}$ (m1) $\frac{1}{3} \cdot \sqrt{6}$ (m1)
f' signs: $(-\infty, \frac{-1}{3} \cdot \sqrt{6}) - (\frac{-1}{3} \cdot \sqrt{6}, \frac{1}{3} \cdot \sqrt{6}) + (\frac{1}{3} \cdot \sqrt{6}, +\infty) -$
 $(\frac{-1}{3} \cdot \sqrt{6}, -\frac{8}{9} \cdot \sqrt{6} - 2)$ local minimum
 $(\frac{1}{3} \cdot \sqrt{6}, \frac{8}{9} \cdot \sqrt{6} - 2)$ local maximum
Increasing: $(\frac{-1}{3} \cdot \sqrt{6}, \frac{1}{3} \cdot \sqrt{6})$
Decreasing: $(-\infty, \frac{-1}{3} \cdot \sqrt{6}) (\frac{1}{3} \cdot \sqrt{6}, +\infty)$

6. Differentiate and analyze signs.

$f(x) = 2 \cdot x^4 - 8 \cdot x^3 + 4 \cdot x^2 + 8 \cdot x - 10$
 $f'(x) = 8 \cdot x^3 - 24 \cdot x^2 + 8 \cdot x + 8$
Zeros: $-\sqrt{2} + 1$ (m1) 1 (m1) $\sqrt{2} + 1$ (m1)
f' signs: $(-\infty, -\sqrt{2} + 1) - (-\sqrt{2} + 1, 1) + (1, \sqrt{2} + 1) - (\sqrt{2} + 1, +\infty) +$
 $(-\sqrt{2} + 1, -12)$ local minimum
 $(1, -4)$ local maximum
 $(\sqrt{2} + 1, -12)$ local minimum
Increasing: $(-\sqrt{2} + 1, 1) (\sqrt{2} + 1, +\infty)$
Decreasing: $(-\infty, -\sqrt{2} + 1) (1, \sqrt{2} + 1)$

7. Differentiate and analyze signs.

$f(x) = x^2 + 4 \cdot x + 7$
 $f'(x) = 2 \cdot x + 4$
Zeros: -2 (m1)
f' signs: $(-\infty, -2) - (-2, +\infty) +$
 $(-2, 3)$ local minimum
Increasing: $(-2, +\infty)$
Decreasing: $(-\infty, -2)$

8. Differentiate and analyze signs.

$f(x) = -x^3 + 6 \cdot x^2 - 12 \cdot x + 7$
 $f'(x) = -3 \cdot x^2 + 12 \cdot x - 12$
Zeros: 2 (m2)
f' signs: $(-\infty, 2) - (2, +\infty) -$
 $(2, -1)$ stationary non extremum
Increasing: none
Decreasing: $(-\infty, +\infty)$
Same derivative sign on both sides: stationary, but not an extremum.