

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Concavity

Find the actual inflection coordinates and give the maximal open intervals where the polynomial is concave up and concave down.

1. Find actual inflection points; give concave-up/down intervals.

$$f(x) = 2 \cdot x^2 + 4$$

2. Find actual inflection points; give concave-up/down intervals.

$$f(x) = -2 \cdot x^3 - 12 \cdot x^2 - 18 \cdot x - 3$$

3. Find actual inflection points; give concave-up/down intervals.

$$f(x) = -2 \cdot x^4 - 8 \cdot x^3 + 16 \cdot x + 13$$

4. Find actual inflection points; give concave-up/down intervals.

$$f(x) = x^2 - 4 \cdot x$$

Concavity

Find the actual inflection coordinates and give the maximal open intervals where the polynomial is concave up and concave down.

5. Find actual inflection points; give concave-up/down intervals.

$f(x) = -x^3 + 2 \cdot x - 1$

6. Find actual inflection points; give concave-up/down intervals.

$f(x) = -2 \cdot x^4 + 8 \cdot x^3 - 4 \cdot x^2 - 8 \cdot x + 3$

7. Find actual inflection points; give concave-up/down intervals.

$f(x) = -x^2 + 4 \cdot x - 1$

8. Find actual inflection points; give concave-up/down intervals.

$f(x) = -2 \cdot x^3 + 2$

Concavity
Worked Answer Key

Find the actual inflection coordinates and give the maximal open intervals where the polynomial is concave up and concave down.

1. Differentiate and analyze signs.

$f(x) = 2 \cdot x^2 + 4$

$f'(x) = 4 \cdot x$

$f''(x) = 4$

f'' has no real zeros.

f'' signs: $(-\infty, +\infty) +$

No points.

Concave up: $(-\infty, +\infty)$

Concave down: none

2. Differentiate and analyze signs.

$f(x) = -2 \cdot x^3 - 12 \cdot x^2 - 18 \cdot x - 3$

$f'(x) = -6 \cdot x^2 - 24 \cdot x - 18$

$f''(x) = -12 \cdot x - 24$

Zeros: -2 (m1)

f'' signs: $(-\infty, -2) +$ $(-2, +\infty) -$

$(-2, 1)$ inflection point

Concave up: $(-\infty, -2)$

Concave down: $(-2, +\infty)$

3. Differentiate and analyze signs.

$f(x) = -2 \cdot x^4 - 8 \cdot x^3 + 16 \cdot x + 13$

$f'(x) = -8 \cdot x^3 - 24 \cdot x^2 + 16$

$f''(x) = -24 \cdot x^2 - 48 \cdot x$

Zeros: -2 (m1) 0 (m1)

f'' signs: $(-\infty, -2) -$ $(-2, 0) +$ $(0, +\infty) -$

$(-2, 13)$ inflection point

$(0, 13)$ inflection point

Concave up: $(-2, 0)$

Concave down: $(-\infty, -2)$ $(0, +\infty)$

4. Differentiate and analyze signs.

$f(x) = x^2 - 4 \cdot x$

$f'(x) = 2 \cdot x - 4$

$f''(x) = 2$

f'' has no real zeros.

f'' signs: $(-\infty, +\infty) +$

No points.

Concave up: $(-\infty, +\infty)$

Concave down: none

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Name _____ Score _____ Teacher _____ Date _____

Concavity

Worked Answer Key

Find the actual inflection coordinates and give the maximal open intervals where the polynomial is concave up and concave down.

5. Differentiate and analyze signs.

$$f(x) = -x^3 + 2 \cdot x - 1$$

$$f'(x) = -3 \cdot x^2 + 2$$

$$f''(x) = -6 \cdot x$$

$$\text{Zeros: } 0 \text{ (m1)}$$

$$f'' \text{ signs: } (-\infty, 0) + (0, +\infty) -$$

$$(0, -1) \text{ inflection point}$$

$$\text{Concave up: } (-\infty, 0)$$

$$\text{Concave down: } (0, +\infty)$$

6. Differentiate and analyze signs.

$$f(x) = -2 \cdot x^4 + 8 \cdot x^3 - 4 \cdot x^2 - 8 \cdot x + 3$$

$$f'(x) = -8 \cdot x^3 + 24 \cdot x^2 - 8 \cdot x - 8$$

$$f''(x) = -24 \cdot x^2 + 48 \cdot x - 8$$

$$\text{Zeros: } -\frac{1}{3} \cdot \sqrt{6} + 1 \text{ (m1)} \quad \frac{1}{3} \cdot \sqrt{6} + 1 \text{ (m1)}$$

$$f'' \text{ signs: } (-\infty, -\frac{1}{3} \cdot \sqrt{6} + 1) - (-\frac{1}{3} \cdot \sqrt{6} + 1, \frac{1}{3} \cdot \sqrt{6} + 1) +$$

$$(\frac{1}{3} \cdot \sqrt{6} + 1, +\infty) -$$

$$(-\frac{1}{3} \cdot \sqrt{6} + 1, \frac{13}{9}) \text{ inflection point}$$

$$(\frac{1}{3} \cdot \sqrt{6} + 1, \frac{13}{9}) \text{ inflection point}$$

$$\text{Concave up: } (-\frac{1}{3} \cdot \sqrt{6} + 1, \frac{1}{3} \cdot \sqrt{6} + 1)$$

$$\text{Concave down: } (-\infty, -\frac{1}{3} \cdot \sqrt{6} + 1) \quad (\frac{1}{3} \cdot \sqrt{6} + 1, +\infty)$$

7. Differentiate and analyze signs.

$$f(x) = -x^2 + 4 \cdot x - 1$$

$$f'(x) = -2 \cdot x + 4$$

$$f''(x) = -2$$

$$f'' \text{ has no real zeros.}$$

$$f'' \text{ signs: } (-\infty, +\infty) -$$

$$\text{No points.}$$

$$\text{Concave up: none}$$

$$\text{Concave down: } (-\infty, +\infty)$$

8. Differentiate and analyze signs.

$$f(x) = -2 \cdot x^3 + 2$$

$$f'(x) = -6 \cdot x^2$$

$$f''(x) = -12 \cdot x$$

$$\text{Zeros: } 0 \text{ (m1)}$$

$$f'' \text{ signs: } (-\infty, 0) + (0, +\infty) -$$

$$(0, 2) \text{ inflection point}$$

$$\text{Concave up: } (-\infty, 0)$$

$$\text{Concave down: } (0, +\infty)$$