Derivative from the Definition

Use the limit definition and show cancellation or a standard-limit identity.

1. y = (x^2+1)/(x-2)

2. y = 1(3x^2+1)^5

3. y = (x^2-6x)/(2x^2-12x+18)

4. y = (1/2)/(x^2-4x+4)+(-3)/(x^2)

5. y = 4sin(-6x+3)

Derivative from the Definition

Use the limit definition and show cancellation or a standard-limit identity.

6. $y = 2x^3-x+4$

7. $y = -2\cos(3x-12)$

8. $y = x^4-2x^2+3$

9. $y = -x^2-3x-5$

10. $y = 1(-x^3-x)^{-3}$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

1. y = (x^2+1)/(x-2)

lim h->0 [(((x+h)^2+1)/((x+h)-2))-((x^2+1)/(x-2))]/h

[f(x+h)-f(x)]/h = [(((x+h)^2+1)/((x+h)-2))-((x^2+1)/(x-2))]/h.

N1(x)=x^2+1, D1(x)=x-2: [N1(x+h)D1(x)-N1(x)D1(x+h)]/[hD1(x+h)D1(x)] = [(((x+h)^2+1)-(x^2+1))/h)(x-2)-(x^2+1)((((x+h)-2)-(x-2))/h)]/[((x+h)-2)(x-2)].

For h != 0 the canceled expression is [(Delta_h N1)D1(x)-N1(x)(Delta_h D1)]/[D1(x+h)D1(x)].

Taking h -> 0 gives [(2x)(x-2)-(x^2+1)(1)]/(x-2)^2.

dy/dx = [(2x)(x-2)-(x^2+1)(1)]/(x-2)^2

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

2. $y = 1(3x^2+1)^5$

$\lim_{h\to 0} [(1(3(x+h)^2+1)^5)-(1(3x^2+1)^5)]/h$

$[f(x+h)-f(x)]/h = [(1(3(x+h)^2+1)^5)-(1(3x^2+1)^5)]/h.$

Let $A=3(x+h)^2+1$, $B=3x^2+1$. $A^5-B^5=(A-B)(A^4+A^3B+...+B^4)$; $(A-B)/h=[(3(x+h)^2+1)-(3x^2+1)]/h.$

For $h \neq 0$ the canceled expression is $1([(3(x+h)^2+1)-(3x^2+1)]/h)(A^4+...+B^4).$

Taking $h \to 0$ gives $5(3x^2+1)^4(6x).$

$dy/dx = 5(3x^2+1)^4(6x)$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

3. y = (x^2-6x)/(2x^2-12x+18)

lim h->0 [(((x+h)^2-6(x+h))/(2(x+h)^2-12(x+h)+18))-((x^2-6x)/(2x^2-12x+18))]/h

[f(x+h)-f(x)]/h = [(((x+h)^2-6(x+h))/(2(x+h)^2-12(x+h)+18))-((x^2-6x)/(2x^2-12x+18))]/h.

N1(x)=x^2-6x, D1(x)=2x^2-12x+18: [N1(x+h)D1(x)-N1(x)D1(x+h)]/[hD1(x+h)D1(x)] = [(((x+h)^2-6(x+h))-(x^2-6x))/h)(2x^2-12x+18)-(x^2-6x)(((2(x+h)^2-12(x+h)+18)-(2x^2-12x+18))/h)]/[(2(x+h)^2-12(x+h)+18)(2x^2-12x+18)].

For h != 0 the canceled expression is [(Delta_h N1)D1(x)-N1(x)(Delta_h D1)]/[D1(x+h)D1(x)].

Taking h -> 0 gives [(2x-6)(2x^2-12x+18)-(x^2-6x)(4x-12)]/(2x^2-12x+18)^2.

dy/dx = [(2x-6)(2x^2-12x+18)-(x^2-6x)(4x-12)]/(2x^2-12x+18)^2

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

4. y = (1/2)/(x^2-4x+4)+(-3)/(x^2)

lim h->0 [((1/2)/((x+h)^2-4(x+h)+4)+(-3)/((x+h)^2))-((1/2)/(x^2-4x+4)+(-3)/(x^2))]/h

[f(x+h)-f(x)]/h = [((1/2)/((x+h)^2-4(x+h)+4)+(-3)/((x+h)^2))-((1/2)/(x^2-4x+4)+(-3)/(x^2))]/h.

N1(x)=1/2, D1(x)=x^2-4x+4: [N1(x+h)D1(x)-N1(x)D1(x+h)]/[hD1(x+h)D1(x)] = [(((1/2)-(1/2))/h)(x^2-4x+4)-(1/2)((((x+h)^2-4(x+h)+4)-(x^2-4x+4))/h)]/[((x+h)^2-4(x+h)+4)(x^2-4x+4)]. N2(x)=-3, D2(x)=x^2: [N2(x+h)D2(x)-N2(x)D2(x+h)]/[hD2(x+h)D2(x)] = [(((-3)-(-3))/h)(x^2)-(-3)((((x+h)^2)-(x^2))/h)]/[((x+h)^2)(x^2)].

For h != 0 the canceled expression is [(Delta_h N1)D1(x)-N1(x)(Delta_h D1)]/[D1(x+h)D1(x)] + [(Delta_h N2)D2(x)-N2(x)(Delta_h D2)]/[D2(x+h)D2(x)].

Taking h -> 0 gives [(0)(x^2-4x+4)-(1/2)(2x-4)]/(x^2-4x+4)^2+[(0)(x^2)-(-3)(2x)]/(x^2)^2.

dy/dx = [(0)(x^2-4x+4)-(1/2)(2x-4)]/(x^2-4x+4)^2+[(0)(x^2)-(-3)(2x)]/(x^2)^2

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

5. y = 4sin(-6x+3)

lim h->0 [(4sin(-6(x+h)+3))-(4sin(-6x+3))]/h

[f(x+h)-f(x)]/h = [(4sin(-6(x+h)+3))-(4sin(-6x+3))]/h.

sin(-6x+3-6h)-sin(-6x+3)=sin(-6x+3)(cos(-6h)-1)+cos(-6x+3)sin(-6h).

For h != 0 the canceled expression is 4[sin(-6x+3)(cos(-6h)-1)+cos(-6x+3)sin(-6h)]/h.

Taking h -> 0 gives -24cos(-6x+3).

sin(t)/t -> 1

(cos(t)-1)/t -> 0

dy/dx = -24cos(-6x+3)

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

6. $y = 2x^3-x+4$

$\lim_{h\to 0} [(2(x+h)^3-(x+h)+4)-(2x^3-x+4)]/h$

$[f(x+h)-f(x)]/h = [(2(x+h)^3-(x+h)+4)-(2x^3-x+4)]/h$.

For every $k>=1$, $(x+h)^k-x^k=h[(x+h)^{(k-1)}+(x+h)^{(k-2)}x+...+x^{(k-1)}]$. Apply this identity to $2(x+h)^3-(x+h)+4-(2x^3-x+4)$.

For $h \neq 0$ the canceled expression is $[(2(x+h)^3-(x+h)+4)-(2x^3-x+4)]/h$ after the displayed power-difference cancellation.

Taking $h \to 0$ gives $6x^2-1$.

$dy/dx = 6x^2-1$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

7. y = -2cos(3x-12)

lim h->0 [(-2cos(3(x+h)-12))-(-2cos(3x-12))]/h

[f(x+h)-f(x)]/h = [(-2cos(3(x+h)-12))-(-2cos(3x-12))]/h.

cos(3x-12+3h)-cos(3x-12)=cos(3x-12)(cos(3h)-1)-sin(3x-12)sin(3h).

For h != 0 the canceled expression is -2[cos(3x-12)(cos(3h)-1)-sin(3x-12)sin(3h)]/h.

Taking h -> 0 gives 6sin(3x-12).

sin(t)/t -> 1

(cos(t)-1)/t -> 0

dy/dx = 6sin(3x-12)

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

8. $y = x^4-2x^2+3$

$\lim_{h\to 0} [((x+h)^4-2(x+h)^2+3)-(x^4-2x^2+3)]/h$

$[f(x+h)-f(x)]/h = [((x+h)^4-2(x+h)^2+3)-(x^4-2x^2+3)]/h$.

For every $k>=1$, $(x+h)^k-x^k=h[(x+h)^{(k-1)}+(x+h)^{(k-2)}x+...+x^{(k-1)}]$. Apply this identity to $(x+h)^4-2(x+h)^2+3-(x^4-2x^2+3)$.

For $h \neq 0$ the canceled expression is $[((x+h)^4-2(x+h)^2+3)-(x^4-2x^2+3)]/h$ after the displayed power-difference cancellation.

Taking $h \to 0$ gives $4x^3-4x$.

$dy/dx = 4x^3-4x$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

9. $y = -x^2-3x-5$

$\lim_{h\to 0} [(-(x+h)^2-3(x+h)-5)-(-x^2-3x-5)]/h$

$[f(x+h)-f(x)]/h = [(-(x+h)^2-3(x+h)-5)-(-x^2-3x-5)]/h$.

For every $k>=1$, $(x+h)^k-x^k=h[(x+h)^{(k-1)}+(x+h)^{(k-2)}x+...+x^{(k-1)}]$. Apply this identity to $-(x+h)^2-3(x+h)-5-(-x^2-3x-5)$.

For $h \neq 0$ the canceled expression is $[(-(x+h)^2-3(x+h)-5)-(-x^2-3x-5)]/h$ after the displayed power-difference cancellation.

Taking $h \to 0$ gives $-2x-3$.

$dy/dx = -2x-3$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

10. $y = 1(-x^3-x)^{-3}$

$\lim_{h\to 0} [(1(-(x+h)^3-(x+h))^{-3})-(1(-x^3-x)^{-3})]/h$

$[f(x+h)-f(x)]/h = [(1(-(x+h)^3-(x+h))^{-3})-(1(-x^3-x)^{-3})]/h.$

Let $A=-(x+h)^3-(x+h)$, $B=-x^3-x$. $A^{-3}-B^{-3}=(B^3-A^3)/(A^3B^3)=-(A-B)(A^2+A^1B+...+B^2)/(A^3B^3)$;
$(A-B)/h=[(-(x+h)^3-(x+h))-(-x^3-x)]/h.$

For $h \neq 0$ the canceled expression is $-1([(-(x+h)^3-(x+h))-(-x^3-x)]/h)(A^2+...+B^2)/(A^3B^3)$.

Taking $h \to 0$ gives $-3(-x^3-x)^{-4}(-3x^2-1)$.

$dy/dx = -3(-x^3-x)^{-4}(-3x^2-1)$