

Derivative from the Definition

Use the limit definition and show cancellation or a standard-limit identity.

1. $y = (x^2+1)/(x-2)$

2. $y = 1(3x^2+1)^5$

3. $y = (x^2-6x)/(2x^2-12x+18)$

4. $y = (1/2)/(x^2-4x+4)+(-3)/(x^2)$

5. $y = 4\sin(-6x+3)$

Derivative from the Definition

Use the limit definition and show cancellation or a standard-limit identity.

6. $y = 2x^3 - x + 4$

7. $y = -2\cos(3x - 12)$

8. $y = x^4 - 2x^2 + 3$

9. $y = -x^2 - 3x - 5$

10. $y = 1(-x^3 - x)^{-3}$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

1. $y = (x^2+1)/(x-2)$

$$\lim_{h \rightarrow 0} [(((x+h)^2+1)/((x+h)-2)) - ((x^2+1)/(x-2))]/h$$

$$[f(x+h)-f(x)]/h = [(((x+h)^2+1)/((x+h)-2)) - ((x^2+1)/(x-2))]/h.$$

$$N1(x)=x^2+1, D1(x)=x-2: [N1(x+h)D1(x)-N1(x)D1(x+h)]/[hD1(x+h)D1(x)] =$$

$$[(((x+h)^2+1)-(x^2+1))/h(x-2)-(x^2+1)((x+h)-2)/(x-2))/h]/[(x+h)-2(x-2)].$$

$$\text{For } h \neq 0 \text{ the canceled expression is } [(\Delta_h N1)D1(x)-N1(x)(\Delta_h D1)]/[D1(x+h)D1(x)].$$

$$\text{Taking } h \rightarrow 0 \text{ gives } [(2x)(x-2)-(x^2+1)(1)]/(x-2)^2.$$

$$dy/dx = [(2x)(x-2)-(x^2+1)(1)]/(x-2)^2$$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

2. $y = 1(3x^2+1)^5$

$$\lim_{h \rightarrow 0} \frac{1(3(x+h)^2+1)^5 - 1(3x^2+1)^5}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{1(3(x+h)^2+1)^5 - 1(3x^2+1)^5}{h}$$

Let $A=3(x+h)^2+1$, $B=3x^2+1$. $A^5 - B^5 = (A-B)(A^4 + A^3B + \dots + B^4)$; $(A-B)/h = [(3(x+h)^2+1) - (3x^2+1)]/h$.

For $h \neq 0$ the canceled expression is $1([(3(x+h)^2+1) - (3x^2+1)]/h)(A^4 + \dots + B^4)$.

Taking $h \rightarrow 0$ gives $5(3x^2+1)^4(6x)$.

$$dy/dx = 5(3x^2+1)^4(6x)$$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

3. $y = (x^2 - 6x)/(2x^2 - 12x + 18)$

$$\lim_{h \rightarrow 0} \frac{(((x+h)^2 - 6(x+h))/(2(x+h)^2 - 12(x+h) + 18)) - ((x^2 - 6x)/(2x^2 - 12x + 18))}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(((x+h)^2 - 6(x+h))/(2(x+h)^2 - 12(x+h) + 18)) - ((x^2 - 6x)/(2x^2 - 12x + 18))}{h}$$

$$N1(x) = x^2 - 6x, D1(x) = 2x^2 - 12x + 18: [N1(x+h)D1(x) - N1(x)D1(x+h)]/[hD1(x+h)D1(x)] =$$

$$\frac{(((x+h)^2 - 6(x+h)) - (x^2 - 6x))/h \cdot (2x^2 - 12x + 18) - (x^2 - 6x)((2(x+h)^2 - 12(x+h) + 18) - (2x^2 - 12x + 18))/h}{((2(x+h)^2 - 12(x+h) + 18)(2x^2 - 12x + 18))}$$

$$\text{For } h \neq 0 \text{ the canceled expression is } \frac{(\Delta_h N1)D1(x) - N1(x)(\Delta_h D1)}{[D1(x+h)D1(x)]}$$

$$\text{Taking } h \rightarrow 0 \text{ gives } \frac{(2x-6)(2x^2-12x+18) - (x^2-6x)(4x-12)}{(2x^2-12x+18)^2}$$

$$dy/dx = \frac{(2x-6)(2x^2-12x+18) - (x^2-6x)(4x-12)}{(2x^2-12x+18)^2}$$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

4. $y = (1/2)/(x^2-4x+4)+(-3)/(x^2)$

$$\lim_{h \rightarrow 0} \left[\frac{(1/2)/((x+h)^2-4(x+h)+4)+(-3)/((x+h)^2)-((1/2)/(x^2-4x+4)+(-3)/(x^2))}{h} \right]$$

$$[f(x+h)-f(x)]/h = [((1/2)/((x+h)^2-4(x+h)+4)+(-3)/((x+h)^2))-((1/2)/(x^2-4x+4)+(-3)/(x^2))]/h.$$

$$N1(x)=1/2, D1(x)=x^2-4x+4: [N1(x+h)D1(x)-N1(x)D1(x+h)]/[hD1(x+h)D1(x)] =$$

$$[(((1/2)-(1/2))/h)(x^2-4x+4)-(1/2)((x+h)^2-4(x+h)+4)-(x^2-4x+4)/h)]/[((x+h)^2-4(x+h)+4)(x^2-4x+4)].$$

$$N2(x)=-3, D2(x)=x^2: [N2(x+h)D2(x)-N2(x)D2(x+h)]/[hD2(x+h)D2(x)] =$$

$$[(((-3)-(-3))/h)(x^2)-(-3)((x+h)^2-(x^2))/h]/[((x+h)^2)(x^2)].$$

For $h \neq 0$ the canceled expression is $[(\Delta_h N1)D1(x)-N1(x)(\Delta_h D1)]/[D1(x+h)D1(x)] + [(\Delta_h N2)D2(x)-N2(x)(\Delta_h D2)]/[D2(x+h)D2(x)]$.

Taking $h \rightarrow 0$ gives $[(0)(x^2-4x+4)-(1/2)(2x-4)]/(x^2-4x+4)^2 + [(0)(x^2)-(-3)(2x)]/(x^2)^2$.

$$dy/dx = [(0)(x^2-4x+4)-(1/2)(2x-4)]/(x^2-4x+4)^2 + [(0)(x^2)-(-3)(2x)]/(x^2)^2$$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

5. $y = 4\sin(-6x+3)$

$$\lim_{h \rightarrow 0} [(4\sin(-6(x+h)+3)) - (4\sin(-6x+3))]/h$$

$$[f(x+h) - f(x)]/h = [(4\sin(-6(x+h)+3)) - (4\sin(-6x+3))]/h.$$

$$\sin(-6x+3-6h) - \sin(-6x+3) = \sin(-6x+3)(\cos(-6h)-1) + \cos(-6x+3)\sin(-6h).$$

$$\text{For } h \neq 0 \text{ the canceled expression is } 4[\sin(-6x+3)(\cos(-6h)-1) + \cos(-6x+3)\sin(-6h)]/h.$$

$$\text{Taking } h \rightarrow 0 \text{ gives } -24\cos(-6x+3).$$

$$\sin(t)/t \rightarrow 1$$

$$(\cos(t)-1)/t \rightarrow 0$$

$$dy/dx = -24\cos(-6x+3)$$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

6. $y = 2x^3 - x + 4$

$$\lim_{h \rightarrow 0} [(2(x+h)^3 - (x+h) + 4) - (2x^3 - x + 4)]/h$$

$$[f(x+h) - f(x)]/h = [(2(x+h)^3 - (x+h) + 4) - (2x^3 - x + 4)]/h.$$

For every $k \geq 1$, $(x+h)^k - x^k = h[(x+h)^{k-1} + (x+h)^{k-2}x + \dots + x^{k-1}]$. Apply this identity to $2(x+h)^3 - (x+h) + 4 - (2x^3 - x + 4)$.

For $h \neq 0$ the canceled expression is $[(2(x+h)^3 - (x+h) + 4) - (2x^3 - x + 4)]/h$ after the displayed power-difference cancellation.

Taking $h \rightarrow 0$ gives $6x^2 - 1$.

$$dy/dx = 6x^2 - 1$$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

7. $y = -2\cos(3x-12)$

$$\lim_{h \rightarrow 0} \frac{[-2\cos(3(x+h)-12)) - (-2\cos(3x-12))]}{h}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{[-2\cos(3(x+h)-12)) - (-2\cos(3x-12))]}{h}.$$

$$\cos(3x-12+3h)-\cos(3x-12)=\cos(3x-12)(\cos(3h)-1)-\sin(3x-12)\sin(3h).$$

$$\text{For } h \neq 0 \text{ the canceled expression is } -2[\cos(3x-12)(\cos(3h)-1)-\sin(3x-12)\sin(3h)]/h.$$

$$\text{Taking } h \rightarrow 0 \text{ gives } 6\sin(3x-12).$$

$$\sin(t)/t \rightarrow 1$$

$$(\cos(t)-1)/t \rightarrow 0$$

$$dy/dx = 6\sin(3x-12)$$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

8. $y = x^4 - 2x^2 + 3$

$$\lim_{h \rightarrow 0} \frac{[(x+h)^4 - 2(x+h)^2 + 3] - (x^4 - 2x^2 + 3)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{[(x+h)^4 - 2(x+h)^2 + 3] - (x^4 - 2x^2 + 3)}{h}.$$

For every $k \geq 1$, $(x+h)^k - x^k = h[(x+h)^{(k-1)} + (x+h)^{(k-2)}x + \dots + x^{(k-1)}]$. Apply this identity to $(x+h)^4 - 2(x+h)^2 + 3 - (x^4 - 2x^2 + 3)$.

For $h \neq 0$ the canceled expression is $\frac{[(x+h)^4 - 2(x+h)^2 + 3] - (x^4 - 2x^2 + 3)}{h}$ after the displayed power-difference cancellation.

Taking $h \rightarrow 0$ gives $4x^3 - 4x$.

$$dy/dx = 4x^3 - 4x$$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

9. $y = -x^2 - 3x - 5$

$$\lim_{h \rightarrow 0} \frac{(-(x+h)^2 - 3(x+h) - 5) - (-x^2 - 3x - 5)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(-(x+h)^2 - 3(x+h) - 5) - (-x^2 - 3x - 5)}{h}.$$

For every $k \geq 1$, $(x+h)^k - x^k = h[(x+h)^{(k-1)} + (x+h)^{(k-2)}x + \dots + x^{(k-1)}]$. Apply this identity to $-(x+h)^2 - 3(x+h) - 5 - (-x^2 - 3x - 5)$.

For $h \neq 0$ the canceled expression is $\frac{(-(x+h)^2 - 3(x+h) - 5) - (-x^2 - 3x - 5)}{h}$ after the displayed power-difference cancellation.

Taking $h \rightarrow 0$ gives $-2x - 3$.

$$dy/dx = -2x - 3$$

Derivative from the Definition

Answer Key

Use the limit definition and show cancellation or a standard-limit identity.

10. $y = 1(-x^3-x)^{-3}$

$$\lim_{h \rightarrow 0} [(1(-(x+h)^3-(x+h))^{-3}) - (1(-x^3-x)^{-3})]/h$$

$$[f(x+h)-f(x)]/h = [(1(-(x+h)^3-(x+h))^{-3}) - (1(-x^3-x)^{-3})]/h.$$

Let $A = -(x+h)^3-(x+h)$, $B = -x^3-x$. $A^{-3}-B^{-3} = (B^3-A^3)/(A^3B^3) = -(A-B)(A^2+A^1B+...+B^2)/(A^3B^3)$;
 $(A-B)/h = [(-(x+h)^3-(x+h)) - (-x^3-x)]/h.$

For $h \neq 0$ the canceled expression is $-1([(-(x+h)^3-(x+h)) - (-x^3-x)]/h)(A^2+...+B^2)/(A^3B^3).$

Taking $h \rightarrow 0$ gives $-3(-x^3-x)^{-4}(-3x^2-1).$

$$dy/dx = -3(-x^3-x)^{-4}(-3x^2-1)$$