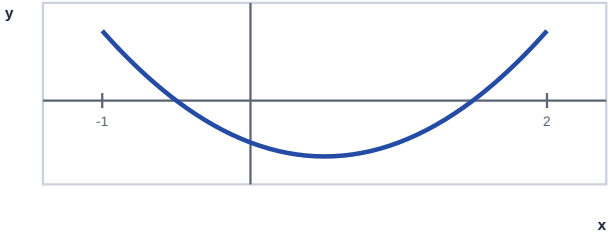


Area Under a Curve

Find total geometric area between the function and the x-axis. Split at every interior zero and add positive segment areas.

1. Find total geometric area on [-1, 2].

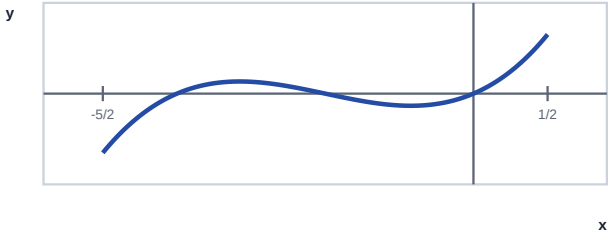
$f(x) = x^2 - x - \frac{3}{4}$



Shade, split at zeros, and add positive areas:

2. Find total geometric area on [-5/2, 1/2].

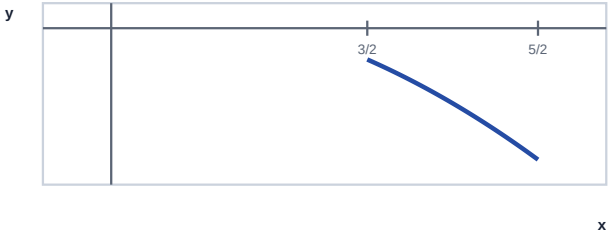
$f(x) = \frac{1}{2} \cdot x^3 + \frac{3}{2} \cdot x^2 + x$



Shade, split at zeros, and add positive areas:

3. Find total geometric area on [3/2, 5/2].

$f(x) = -\frac{1}{2} \cdot x^2 + \frac{1}{2}$



Shade, split at zeros, and add positive areas:

4. Find total geometric area on [7/2, 4].

$f(x) = -\frac{1}{2} \cdot x^3 + 3 \cdot x^2 - \frac{11}{2} \cdot x + 3$



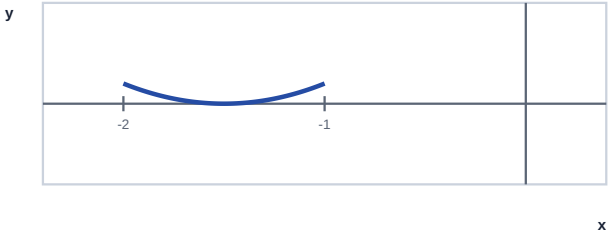
Shade, split at zeros, and add positive areas:

Area Under a Curve

Find total geometric area between the function and the x-axis. Split at every interior zero and add positive segment areas.

5. Find total geometric area on [-2, -1].

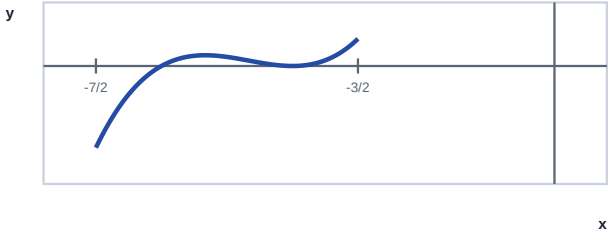
$f(x) = \frac{1}{2} \cdot x^2 + \frac{3}{2} \cdot x + \frac{9}{8}$



Shade, split at zeros, and add positive areas:

6. Find total geometric area on [-7/2, -3/2].

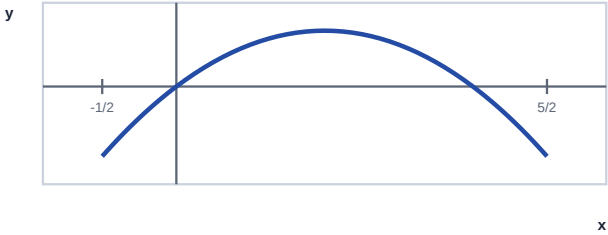
$f(x) = x^3 + 7 \cdot x^2 + 16 \cdot x + 12$



Shade, split at zeros, and add positive areas:

7. Find total geometric area on [-1/2, 5/2].

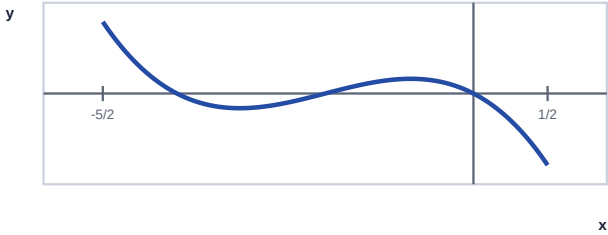
$f(x) = -x^2 + 2 \cdot x$



Shade, split at zeros, and add positive areas:

8. Find total geometric area on [-5/2, 1/2].

$f(x) = -x^3 - 3 \cdot x^2 - 2 \cdot x$



Shade, split at zeros, and add positive areas:

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

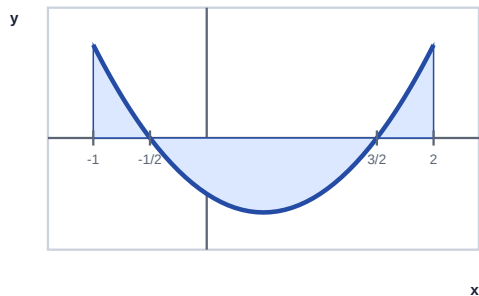
Area Under a Curve

Worked Answer Key

Find total geometric area between the function and the x-axis. Split at every interior zero and add positive segment areas.

1. Zeros/cuts: -1, -1/2, 3/2, 2

$$f(x) = x^2 - x - \frac{3}{4}$$



$$\text{Primitive } F(x) = \frac{1}{3} \cdot x^3 - \frac{1}{2} \cdot x^2 - \frac{3}{4} \cdot x$$

Segment 1: -1 to -1/2; f is positive.

$$\int_{-1}^{-\frac{1}{2}} x^2 - x - \frac{3}{4} dx$$

$$\text{Endpoint subtraction: } \frac{5}{24} - \left(\frac{-1}{12} \right) = \frac{7}{24}$$

Segment 2: -1/2 to 3/2; f is negative.

$$\int_{-\frac{1}{2}}^{\frac{3}{2}} -1 \cdot \left(x^2 - x - \frac{3}{4} \right) dx$$

$$\text{Endpoint subtraction: } -1 \cdot \frac{-9}{8} - \left(-1 \cdot \frac{5}{24} \right) = \frac{4}{3}$$

Segment 3: 3/2 to 2; f is positive.

$$\int_{\frac{3}{2}}^2 x^2 - x - \frac{3}{4} dx$$

$$\text{Endpoint subtraction: } \frac{-5}{6} - \left(\frac{-9}{8} \right) = \frac{7}{24}$$

$$\text{Total area} = \frac{7}{24} + \frac{4}{3} + \frac{7}{24}$$

$$= \frac{23}{12}$$

square units

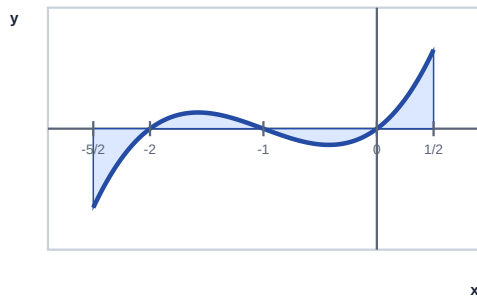
Area Under a Curve

Worked Answer Key

Find total geometric area between the function and the x-axis. Split at every interior zero and add positive segment areas.

2. Zeros/cuts: $-5/2, -2, -1, 0, 1/2$

$$f(x) = \frac{1}{2} \cdot x^3 + \frac{3}{2} \cdot x^2 + x$$



$$\text{Primitive } F(x) = \frac{1}{8} \cdot x^4 + \frac{1}{2} \cdot x^3 + \frac{1}{2} \cdot x^2$$

Segment 1: $-5/2$ to -2 ; f is negative.

$$\int_{-5/2}^{-2} -1 \cdot \left(\frac{1}{2} \cdot x^3 + \frac{3}{2} \cdot x^2 + x \right) dx$$

$$\text{Endpoint subtraction: } -1 \cdot 0 - \left(-1 \cdot \frac{25}{128} \right) = \frac{25}{128}$$

Segment 2: -2 to -1 ; f is positive.

$$\int_{-2}^{-1} \frac{1}{2} \cdot x^3 + \frac{3}{2} \cdot x^2 + x \, dx$$

$$\text{Endpoint subtraction: } \frac{1}{8} - (0) = \frac{1}{8}$$

Segment 3: -1 to 0 ; f is negative.

$$\int_{-1}^0 -1 \cdot \left(\frac{1}{2} \cdot x^3 + \frac{3}{2} \cdot x^2 + x \right) dx$$

$$\text{Endpoint subtraction: } -1 \cdot 0 - \left(-1 \cdot \frac{1}{8} \right) = \frac{1}{8}$$

Segment 4: 0 to $1/2$; f is positive.

$$\int_0^{1/2} \frac{1}{2} \cdot x^3 + \frac{3}{2} \cdot x^2 + x \, dx$$

$$\text{Endpoint subtraction: } \frac{25}{128} - (0) = \frac{25}{128}$$

$$\text{Total area} = \frac{25}{128} + \frac{1}{8} + \frac{1}{8} + \frac{25}{128}$$

$$= \frac{41}{64}$$

square units

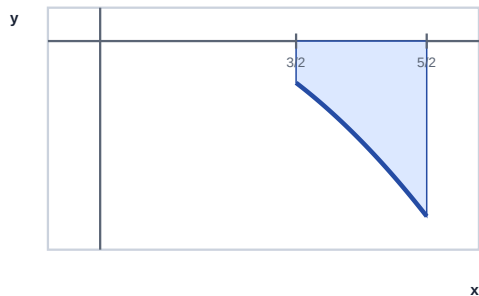
Area Under a Curve

Worked Answer Key

Find total geometric area between the function and the x-axis. Split at every interior zero and add positive segment areas.

3. Zeros/cuts: $\frac{3}{2}$, $\frac{5}{2}$

$$f(x) = -\frac{1}{2} \cdot x^2 + \frac{1}{2}$$



$$\text{Primitive } F(x) = -\frac{1}{6} \cdot x^3 + \frac{1}{2} \cdot x$$

Segment 1: $\frac{3}{2}$ to $\frac{5}{2}$; f is negative.

$$\int_{\frac{3}{2}}^{\frac{5}{2}} -1 \cdot \left(-\frac{1}{2} \cdot x^2 + \frac{1}{2} \right) dx$$

$$\text{Endpoint subtraction: } -1 \cdot \frac{-65}{48} - \left(-1 \cdot \frac{3}{16} \right) = \frac{37}{24}$$

$$\text{Total area} = \frac{37}{24}$$

$$= \frac{37}{24}$$

square units

Area Under a Curve

Worked Answer Key

Find total geometric area between the function and the x-axis. Split at every interior zero and add positive segment areas.

4. Zeros/cuts: 7/2, 4

$$f(x) = -\frac{1}{2} \cdot x^3 + 3 \cdot x^2 - \frac{11}{2} \cdot x + 3$$



$$\text{Primitive } F(x) = -\frac{1}{8} \cdot x^4 + x^3 - \frac{11}{4} \cdot x^2 + 3 \cdot x$$

Segment 1: 7/2 to 4; f is negative.

$$\int_{\frac{7}{2}}^4 -1 \cdot \left(-\frac{1}{2} \cdot x^3 + 3 \cdot x^2 - \frac{11}{2} \cdot x + 3 \right) dx$$

$$\text{Endpoint subtraction: } -1 \cdot 0 - \left(-1 \cdot \frac{119}{128} \right) = \frac{119}{128}$$

$$\text{Total area} = \frac{119}{128}$$

$$= \frac{119}{128}$$

square units

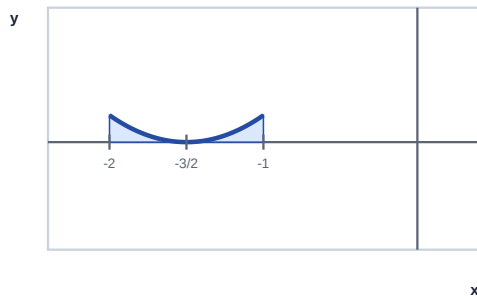
Area Under a Curve

Worked Answer Key

Find total geometric area between the function and the x-axis. Split at every interior zero and add positive segment areas.

5. Zeros/cuts: -2, -3/2, -1

$$f(x) = \frac{1}{2} \cdot x^2 + \frac{3}{2} \cdot x + \frac{9}{8}$$



$$\text{Primitive } F(x) = \frac{1}{6} \cdot x^3 + \frac{3}{4} \cdot x^2 + \frac{9}{8} \cdot x$$

Segment 1: -2 to -3/2; f is positive.

$$\frac{-3}{2} \int_{-2}^{-3/2} \left(\frac{1}{2} \cdot x^2 + \frac{3}{2} \cdot x + \frac{9}{8} \right) dx$$

$$\text{Endpoint subtraction: } \frac{-9}{16} - \left(\frac{-7}{12} \right) = \frac{1}{48}$$

Segment 2: -3/2 to -1; f is positive.

$$\frac{-3}{2} \int_{-3/2}^{-1} \left(\frac{1}{2} \cdot x^2 + \frac{3}{2} \cdot x + \frac{9}{8} \right) dx$$

$$\text{Endpoint subtraction: } \frac{-13}{24} - \left(\frac{-9}{16} \right) = \frac{1}{48}$$

$$\text{Total area} = \frac{1}{48} + \frac{1}{48}$$

$$= \frac{1}{24}$$

square units

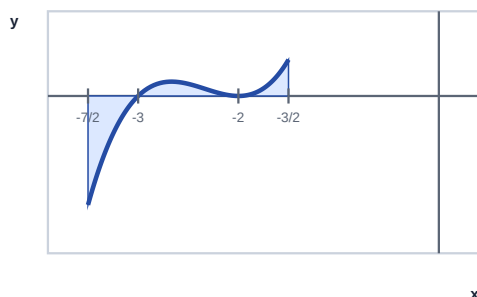
Area Under a Curve

Worked Answer Key

Find total geometric area between the function and the x-axis. Split at every interior zero and add positive segment areas.

6. Zeros/cuts: $-7/2$, -3 , -2 , $-3/2$

$$f(x) = x^3 + 7 \cdot x^2 + 16 \cdot x + 12$$



$$\text{Primitive } F(x) = \frac{1}{4} \cdot x^4 + \frac{7}{3} \cdot x^3 + 8 \cdot x^2 + 12 \cdot x$$

Segment 1: $-7/2$ to -3 ; f is negative.

$$\int_{-7/2}^{-3} -1 \cdot (x^3 + 7 \cdot x^2 + 16 \cdot x + 12) dx$$

$$\text{Endpoint subtraction: } -1 \cdot \frac{-27}{4} - \left(-1 \cdot \frac{-1253}{192} \right) = \frac{43}{192}$$

Segment 2: -3 to -2 ; f is positive.

$$\int_{-3}^{-2} x^3 + 7 \cdot x^2 + 16 \cdot x + 12 dx$$

$$\text{Endpoint subtraction: } \frac{-20}{3} - \left(\frac{-27}{4} \right) = \frac{1}{12}$$

Segment 3: -2 to $-3/2$; f is positive.

$$\int_{-2}^{-3/2} x^3 + 7 \cdot x^2 + 16 \cdot x + 12 dx$$

$$\text{Endpoint subtraction: } \frac{-423}{64} - \left(\frac{-20}{3} \right) = \frac{11}{192}$$

$$\text{Total area} = \frac{43}{192} + \frac{1}{12} + \frac{11}{192}$$

$$= \frac{35}{96}$$

square units

Area Under a Curve

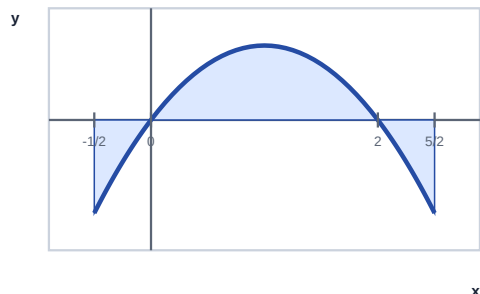
Worked Answer Key

Find total geometric area between the function and the x-axis. Split at every interior zero and add positive segment areas.

7. Zeros/cuts: $-1/2, 0, 2, 5/2$

$$f(x) = -x^2 + 2 \cdot x$$

$$\text{Primitive } F(x) = -\frac{1}{3} \cdot x^3 + x^2$$



Segment 1: $-1/2$ to 0 ; f is negative.

$$\int_{-1/2}^0 -1 \cdot (-x^2 + 2 \cdot x) dx$$

$$\text{Endpoint subtraction: } -1 \cdot 0 - \left(-1 \cdot \frac{7}{24} \right) = \frac{7}{24}$$

Segment 2: 0 to 2 ; f is positive.

$$\int_0^2 -x^2 + 2 \cdot x dx$$

$$\text{Endpoint subtraction: } \frac{4}{3} - (0) = \frac{4}{3}$$

Segment 3: 2 to $5/2$; f is negative.

$$\int_2^{5/2} -1 \cdot (-x^2 + 2 \cdot x) dx$$

$$\text{Endpoint subtraction: } -1 \cdot \frac{25}{24} - \left(-1 \cdot \frac{4}{3} \right) = \frac{7}{24}$$

$$\text{Total area} = \frac{7}{24} + \frac{4}{3} + \frac{7}{24}$$

$$= \frac{23}{12}$$

square units

Area Under a Curve

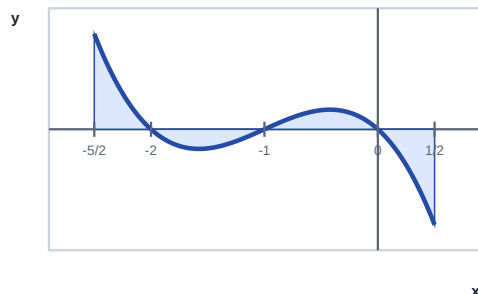
Worked Answer Key

Find total geometric area between the function and the x-axis. Split at every interior zero and add positive segment areas.

8. Zeros/cuts: $-5/2, -2, -1, 0, 1/2$

$$f(x) = -x^3 - 3x^2 - 2x$$

$$\text{Primitive } F(x) = -\frac{1}{4}x^4 - x^3 - x^2$$



Segment 1: $-5/2$ to -2 ; f is positive.

$$\int_{-5/2}^{-2} (-x^3 - 3x^2 - 2x) dx$$

$$\text{Endpoint subtraction: } 0 - \left(\frac{-25}{64} \right) = \frac{25}{64}$$

Segment 2: -2 to -1 ; f is negative.

$$\int_{-2}^{-1} (-x^3 - 3x^2 - 2x) dx$$

$$\text{Endpoint subtraction: } -1 - \frac{-1}{4} - (-1 - 0) = \frac{1}{4}$$

Segment 3: -1 to 0 ; f is positive.

$$\int_{-1}^0 (-x^3 - 3x^2 - 2x) dx$$

$$\text{Endpoint subtraction: } 0 - \left(\frac{-1}{4} \right) = \frac{1}{4}$$

Segment 4: 0 to $1/2$; f is negative.

$$\int_0^{1/2} (-x^3 - 3x^2 - 2x) dx$$

$$\text{Endpoint subtraction: } -1 - \frac{-25}{64} - (-1 - 0) = \frac{25}{64}$$

$$\text{Total area} = \frac{25}{64} + \frac{1}{4} + \frac{1}{4} + \frac{25}{64}$$

$$= \frac{41}{32}$$

square units