

Area Between Curves

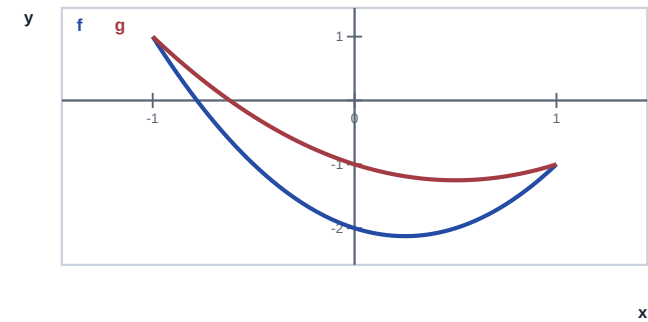
Find the total area between the curves. Split the interval wherever the curve order changes.

1. Integrate with respect to x.

$y = 2 \cdot x^2 - x - 2$

$y = x^2 - x - 1$

Find the intersection limits and all enclosed regions.



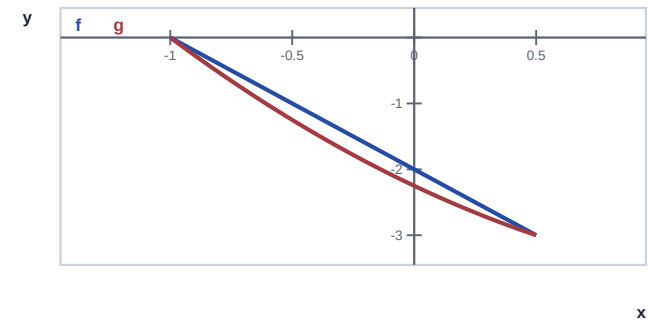
Shade the region, split if needed, and integrate:

2. Integrate with respect to x.

$y = -2 \cdot x - 2$

$y = \frac{1}{2} \cdot x^2 - \frac{7}{4} \cdot x - \frac{9}{4}$

Find the intersection limits and all enclosed regions.



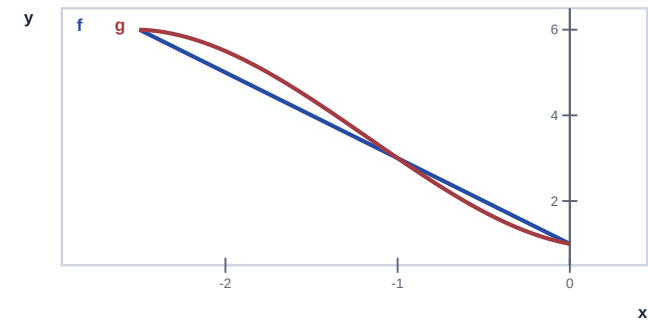
Shade the region, split if needed, and integrate:

3. Integrate with respect to x.

$y = -2 \cdot x + 1$

$y = \frac{1}{2} \cdot x^3 + \frac{7}{4} \cdot x^2 - \frac{3}{4} \cdot x + 1$

Find the intersection limits and all enclosed regions.



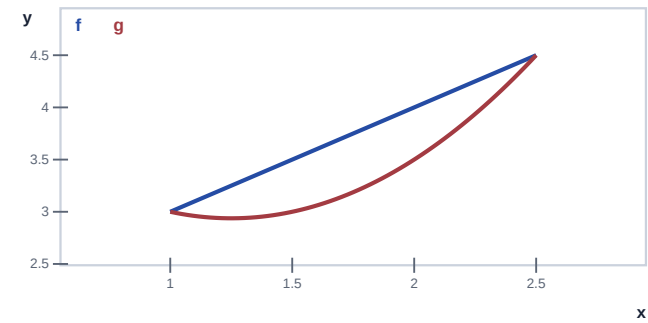
Shade the region, split if needed, and integrate:

4. Integrate with respect to x.

$y = x + 2$

$y = x^2 - \frac{5}{2} \cdot x + \frac{9}{2}$

Find the intersection limits and all enclosed regions.



Shade the region, split if needed, and integrate:

Area Between Curves

Worked Answer Key

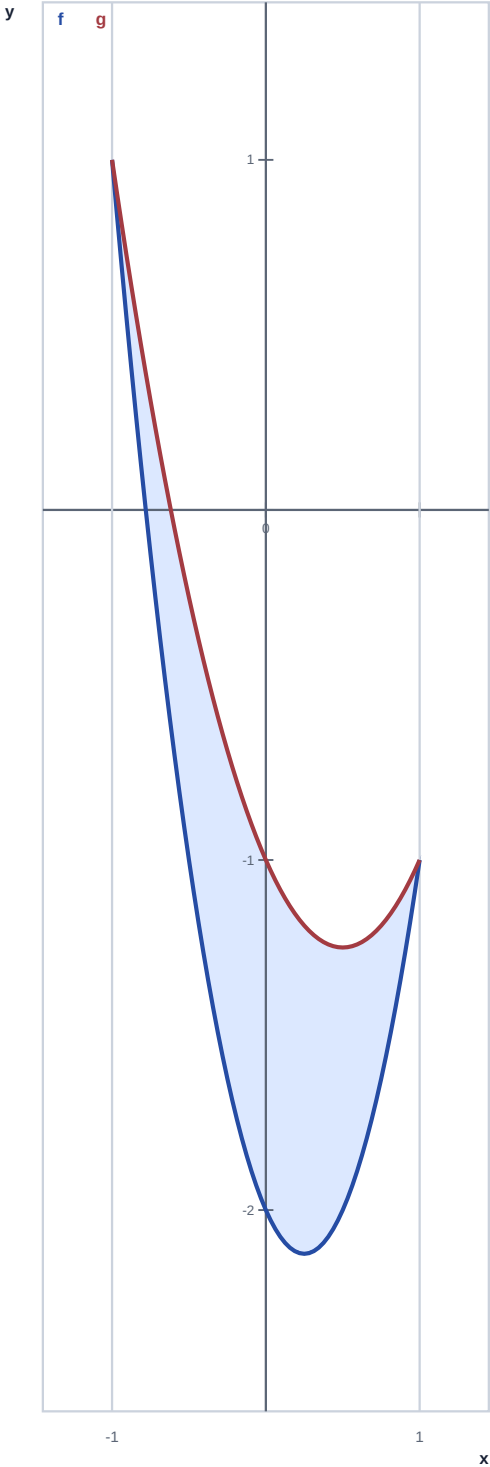
Find the total area between the curves. Split the interval wherever the curve order changes.

1. $y = 2 \cdot x^2 - x - 2$

$g - f = -x^2 + 1 = -1(x + 1)(x - 1)$

Intersections: -1, 1. Bounds: [-1, 1].

$y = x^2 - x - 1$



1) g upper, f lower

$\int_{-1}^1 -x^2 + 1 \, dx$

Primitive: $-\frac{1}{3} \cdot x^3 + x$

Evaluation: $\frac{2}{3} - \left(-\frac{2}{3}\right) = \frac{4}{3}$

Total area: $\frac{4}{3} = \frac{4}{3}$

square units

Area Between Curves

Worked Answer Key

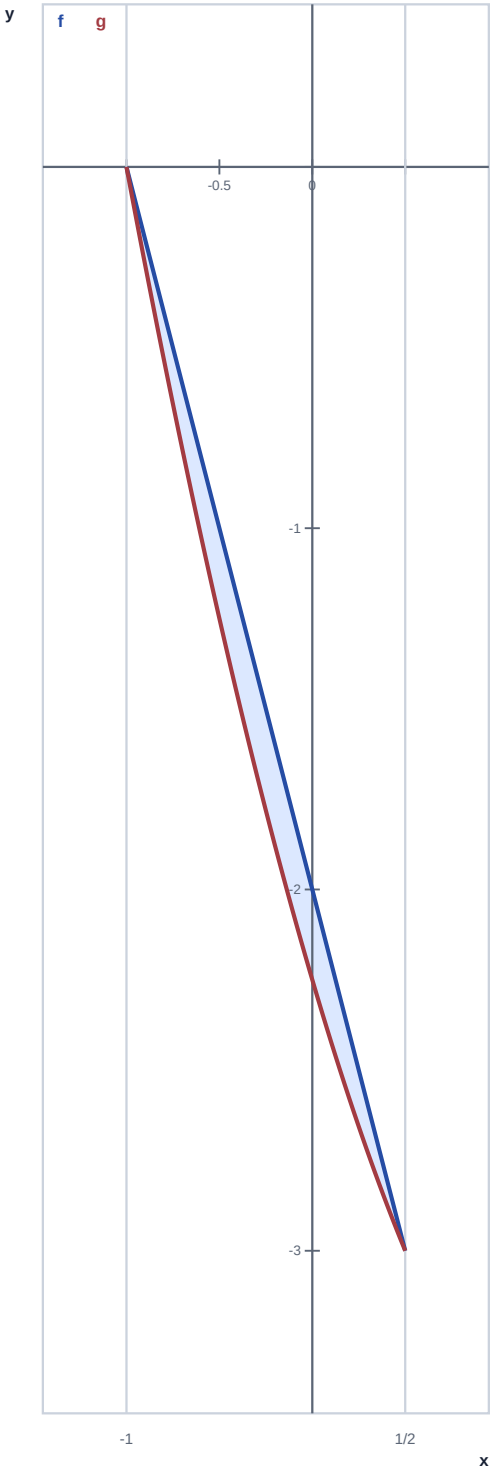
Find the total area between the curves. Split the interval wherever the curve order changes.

2. $y = -2x - 2$

$g - f = 1/2x^2 + 1/4x - 1/4 = 1/2(x + 1)(x - 1/2)$

Intersections: -1, 1/2. Bounds: [-1, 1/2].

$y = \frac{1}{2} \cdot x^2 - \frac{7}{4} \cdot x - \frac{9}{4}$



1) f upper, g lower

$\int_{-1}^{\frac{1}{2}} -\frac{1}{2} \cdot x^2 - \frac{1}{4} \cdot x + \frac{1}{4} dx$

Primitive: $-\frac{1}{6} \cdot x^3 - \frac{1}{8} \cdot x^2 + \frac{1}{4} \cdot x$

Evaluation: $\frac{7}{96} - \left(-\frac{5}{24}\right) = \frac{9}{32}$

Total area: $\frac{9}{32} = \frac{9}{32}$

square units

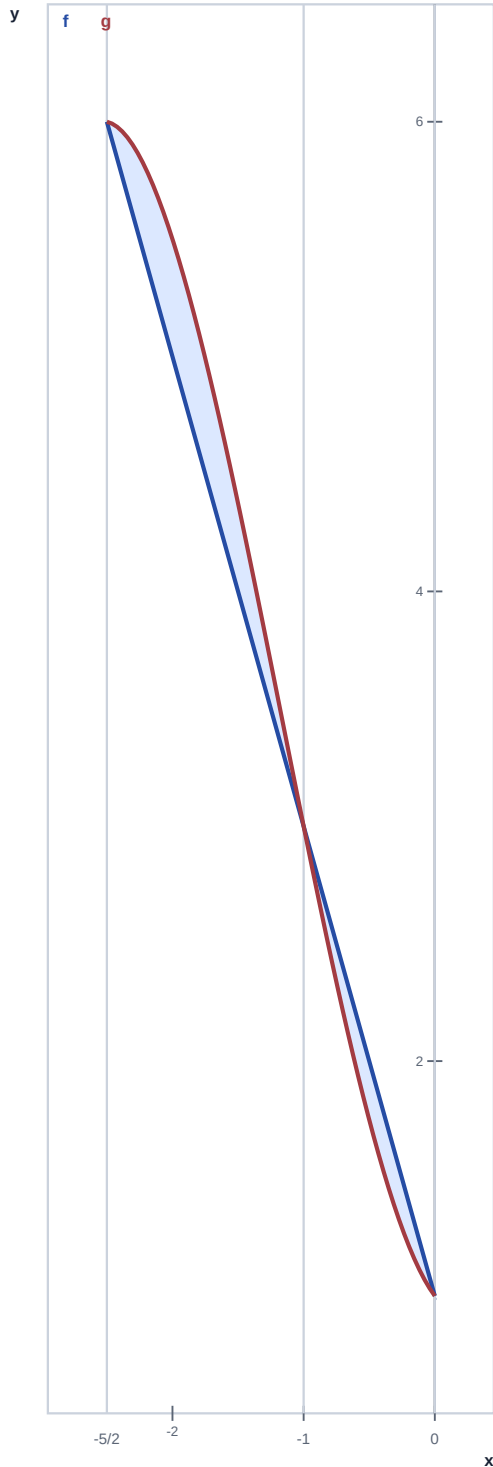
Area Between Curves

Worked Answer Key

Find the total area between the curves. Split the interval wherever the curve order changes.

3. $y = -\frac{1}{2}x^3 + \frac{7}{4}x^2 - \frac{3}{4}x + 1$

$$g - f = \frac{1}{2}x^3 + \frac{7}{4}x^2 + \frac{5}{4}x = \frac{1}{2}(x + \frac{5}{2})(x + 1)(x - 0)$$

Intersections: $-\frac{5}{2}, -1, 0$. Bounds: $[-\frac{5}{2}, 0]$.

1) g upper, f lower

$$\int_{-\frac{5}{2}}^{-1} \left(\frac{1}{2}x^3 + \frac{7}{4}x^2 + \frac{5}{4}x \right) dx$$

$$\text{Primitive: } \frac{1}{8}x^4 + \frac{7}{12}x^3 + \frac{5}{8}x^2$$

$$\text{Evaluation: } \frac{1}{6} - \left(-\frac{125}{384} \right) = \frac{63}{128}$$

2) f upper, g lower

$$\int_{-1}^0 \left(-\frac{1}{2}x^3 - \frac{7}{4}x^2 - \frac{5}{4}x \right) dx$$

$$\text{Primitive: } -\frac{1}{8}x^4 - \frac{7}{12}x^3 - \frac{5}{8}x^2$$

$$\text{Evaluation: } 0 - \left(-\frac{1}{6} \right) = \frac{1}{6}$$

$$\text{Total area: } \frac{63}{128} + \frac{1}{6} = \frac{253}{384}$$

square units

Area Between Curves

Worked Answer Key

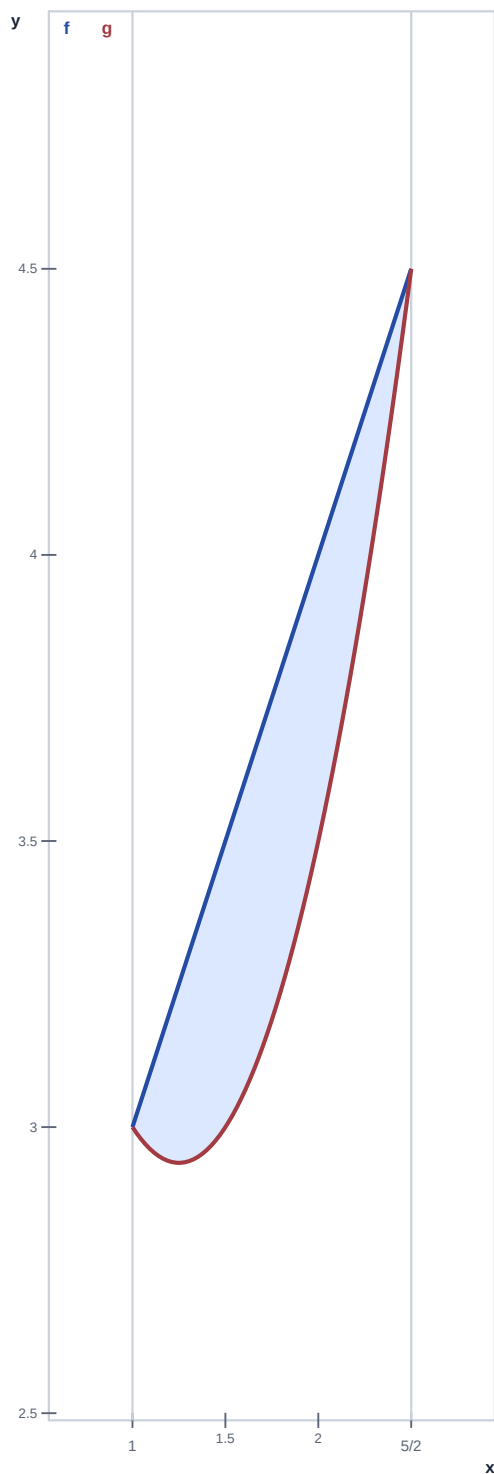
Find the total area between the curves. Split the interval wherever the curve order changes.

4. $y = x + 2$

$g - f = x^2 - 7/2x + 5/2 = 1(x - 1)(x - 5/2)$

Intersections: 1, 5/2. Bounds: [1, 5/2].

$$y = x^2 - \frac{7}{2}x + \frac{9}{2}$$



1) f upper, g lower

$$\int_1^{\frac{5}{2}} -x^2 + \frac{7}{2}x - \frac{5}{2} dx$$

$$\text{Primitive: } -\frac{1}{3}x^3 + \frac{7}{4}x^2 - \frac{5}{2}x$$

$$\text{Evaluation: } -\frac{25}{48} - \left(-\frac{13}{12}\right) = \frac{9}{16}$$

$$\text{Total area: } \frac{9}{16} = \frac{9}{16}$$

square units