

## Area Between Curves

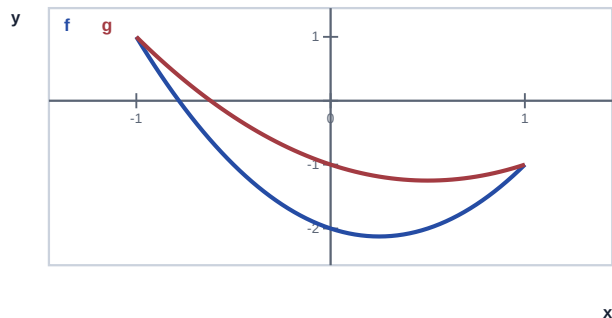
Find the total area between the curves. Split the interval wherever the curve order changes.

### 1. Integrate with respect to x.

$$y = 2 \cdot x^2 - x - 2$$

$$y = x^2 - x - 1$$

Find the intersection limits and all enclosed regions.



Shade the region, split if needed, and integrate:

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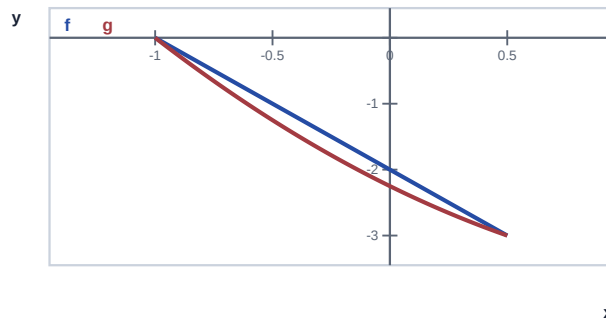
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### 2. Integrate with respect to x.

$$y = -2 \cdot x - 2$$

$$y = \frac{1}{2} \cdot x^2 - \frac{7}{4} \cdot x - \frac{9}{4}$$

Find the intersection limits and all enclosed regions.



Shade the region, split if needed, and integrate:

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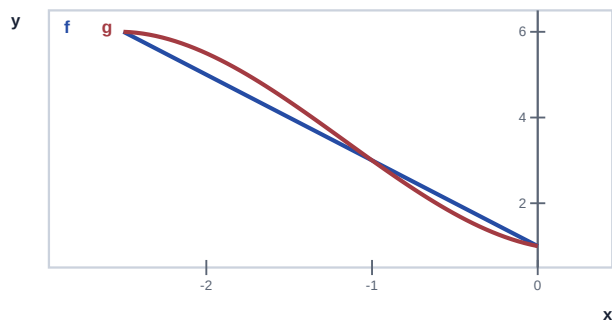
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### 3. Integrate with respect to x.

$$y = -2 \cdot x + 1$$

$$y = \frac{1}{2} \cdot x^3 + \frac{7}{4} \cdot x^2 - \frac{3}{4} \cdot x + 1$$

Find the intersection limits and all enclosed regions.



Shade the region, split if needed, and integrate:

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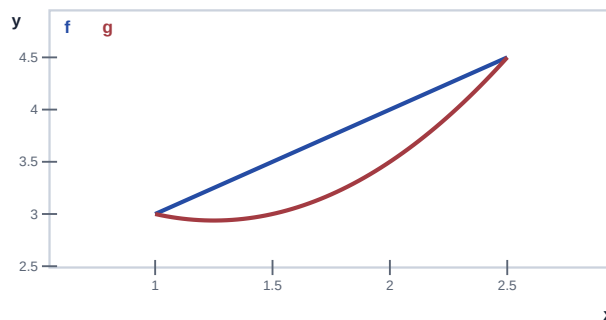
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### 4. Integrate with respect to x.

$$y = x + 2$$

$$y = x^2 - \frac{5}{2} \cdot x + \frac{9}{2}$$

Find the intersection limits and all enclosed regions.



Shade the region, split if needed, and integrate:

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## Area Between Curves

## Worked Answer Key

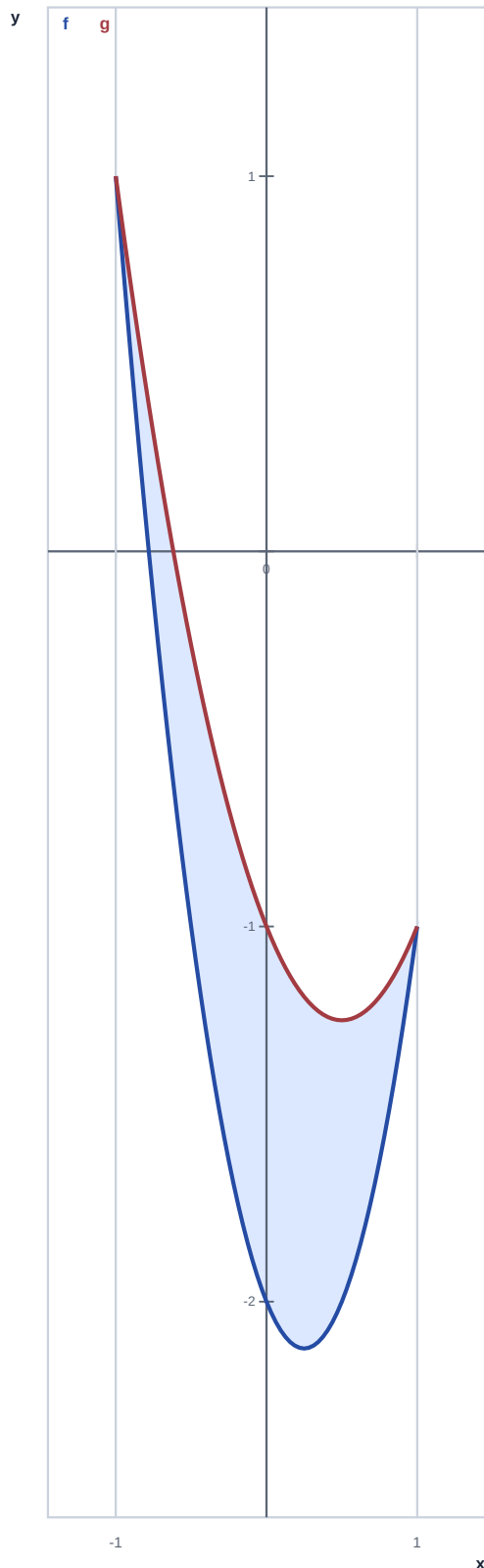
Find the total area between the curves. Split the interval wherever the curve order changes.

1.  $y = 2 \cdot x^2 - x - 2$

$y = x^2 - x - 1$

$g - f = -x^2 + 1 = -(x + 1)(x - 1)$

Intersections: -1, 1. Bounds: [-1, 1].



1) g upper, f lower

$$\int_{-1}^1 -x^2 + 1 \, dx$$

$$\text{Primitive: } -\frac{1}{3}x^3 + x$$

$$\text{Evaluation: } \frac{2}{3} - \left(-\frac{2}{3}\right) = \frac{4}{3}$$

$$\text{Total area: } \frac{4}{3} = \frac{4}{3}$$

square units

## Area Between Curves

## Worked Answer Key

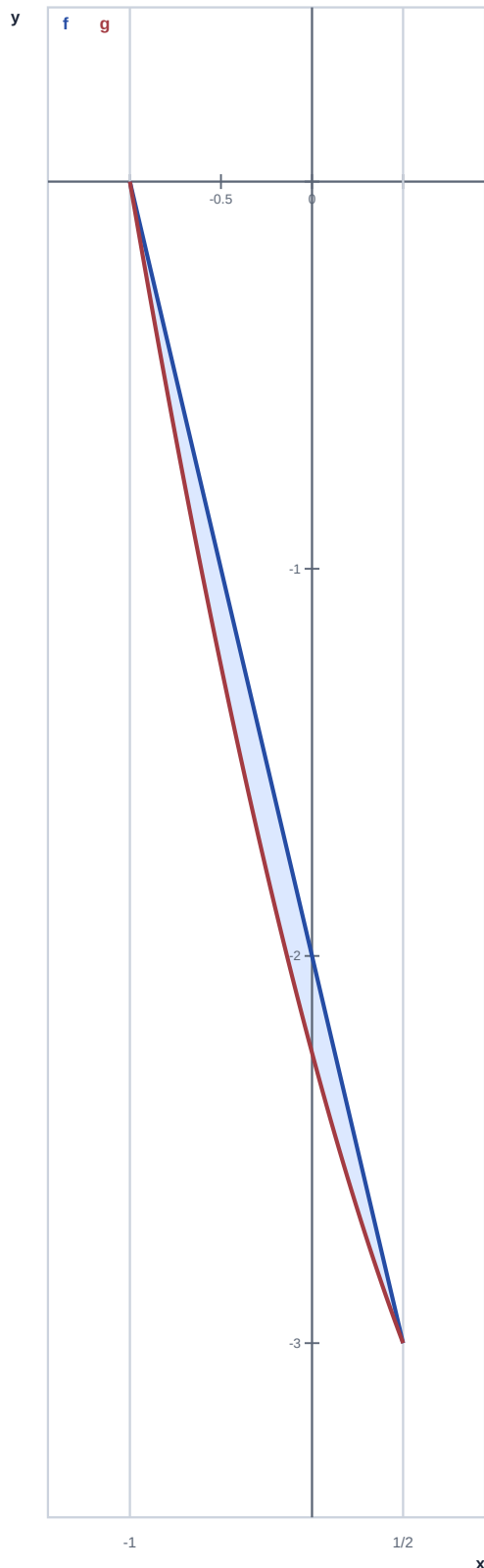
Find the total area between the curves. Split the interval wherever the curve order changes.

2.  $y = -\frac{1}{2}x^2 - \frac{1}{4}x - \frac{1}{4}$

$$g - f = \frac{1}{2}x^2 + \frac{1}{4}x - \frac{1}{4} = \frac{1}{2}(x + 1)(x - \frac{1}{2})$$

Intersections: -1, 1/2. Bounds: [-1, 1/2].

$$y = \frac{1}{2}x^2 - \frac{7}{4}x - \frac{9}{4}$$



1) f upper, g lower

$$\int_{-1}^{\frac{1}{2}} -\frac{1}{2}x^2 - \frac{1}{4}x + \frac{1}{4} dx$$

$$\text{Primitive: } -\frac{1}{6}x^3 - \frac{1}{8}x^2 + \frac{1}{4}x$$

$$\text{Evaluation: } \frac{7}{96} - \left( -\frac{5}{24} \right) = \frac{9}{32}$$

$$\text{Total area: } \frac{9}{32} = \frac{9}{32}$$

square units

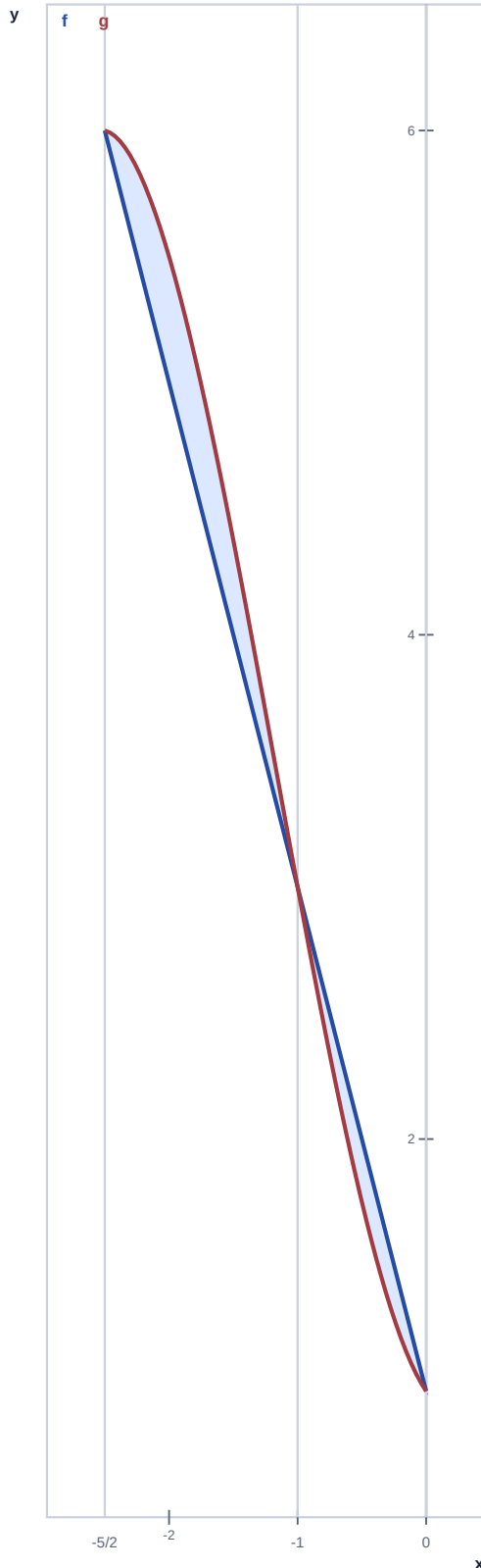
## Area Between Curves

## Worked Answer Key

Find the total area between the curves. Split the interval wherever the curve order changes.

3.  $y = -\frac{1}{2}x^3 + \frac{7}{4}x^2 - \frac{3}{4}x + 1$

$$g - f = \frac{1}{2}x^3 + \frac{7}{4}x^2 + \frac{5}{4}x = \frac{1}{2}(x + 5/2)(x + 1)(x - 0)$$

Intersections:  $-5/2, -1, 0$ . Bounds:  $[-5/2, 0]$ .

1) g upper, f lower

$$\int_{-5/2}^{-1} \left( -\frac{1}{2}x^3 + \frac{7}{4}x^2 - \frac{3}{4}x + 1 - (-x^2) \right) dx$$

$$\text{Primitive: } -\frac{1}{8}x^4 + \frac{7}{12}x^3 - \frac{3}{8}x^2 + x$$

$$\text{Evaluation: } \left( -\frac{1}{8}x^4 + \frac{7}{12}x^3 - \frac{3}{8}x^2 + x \right) \Big|_{-5/2}^{-1} = \frac{63}{128}$$

2) f upper, g lower

$$\int_{-1}^0 \left( -x^2 - \left( -\frac{1}{2}x^3 + \frac{7}{4}x^2 - \frac{3}{4}x + 1 \right) \right) dx$$

$$\text{Primitive: } -\frac{1}{3}x^3 - \frac{1}{2}x^2 + \frac{3}{4}x - 1$$

$$\text{Evaluation: } \left( -\frac{1}{3}x^3 - \frac{1}{2}x^2 + \frac{3}{4}x - 1 \right) \Big|_{-1}^0 = \frac{1}{6}$$

$$\text{Total area: } \frac{63}{128} + \frac{1}{6} = \frac{253}{384}$$

square units

## Area Between Curves

## Worked Answer Key

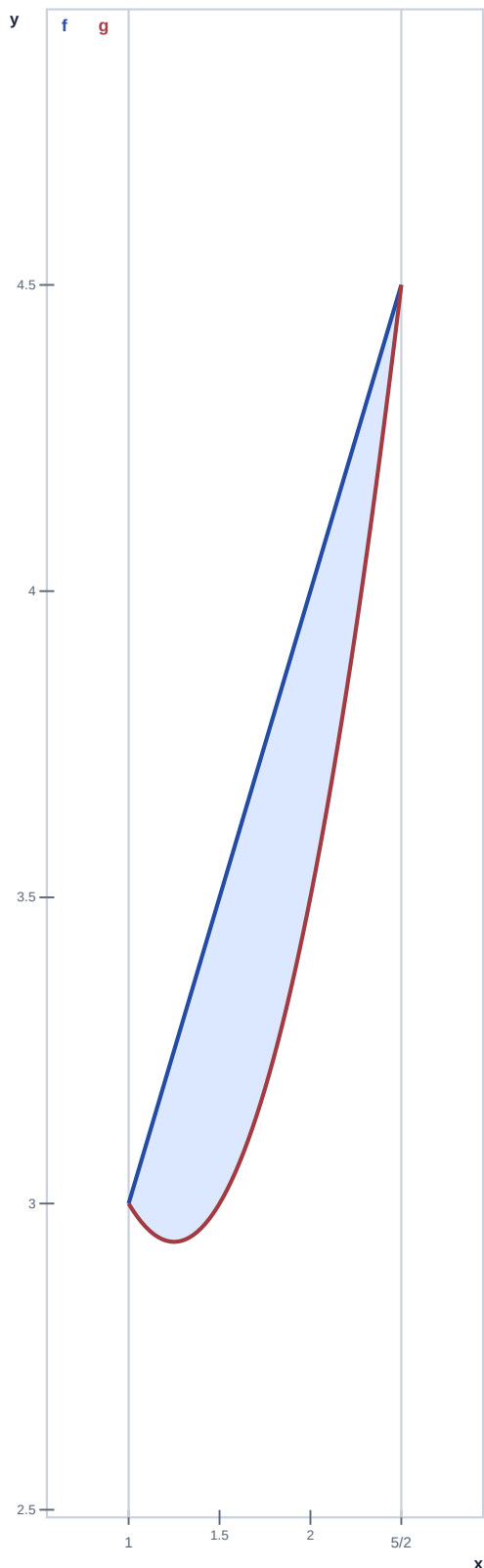
Find the total area between the curves. Split the interval wherever the curve order changes.

4.  $y = x + 2$

$$y = x^2 - \frac{5}{2}x + \frac{9}{2}$$

$$g - f = x^2 - 7/2x + 5/2 = 1(x - 1)(x - 5/2)$$

Intersections: 1, 5/2. Bounds: [1, 5/2].



1) f upper, g lower

$$\int_1^{\frac{5}{2}} -x^2 + \frac{7}{2}x - \frac{5}{2} dx$$

$$\text{Primitive: } -\frac{1}{3}x^3 + \frac{7}{4}x^2 - \frac{5}{2}x$$

$$\text{Evaluation: } -\frac{25}{48} - \left(-\frac{13}{12}\right) = \frac{9}{16}$$

$$\text{Total area: } \frac{9}{16} = \frac{9}{16}$$

square units