

Absolute Extrema

Find the absolute minimum and maximum values on the stated domain, including every input where each value is attained. Explain when an extremum does not exist.

1. Find the absolute minimum and maximum values and all locations where they occur. Explain if either does not exist.

Domain: $\mathbb{R}$ (all real x)

$f(x) = x^2 + 4 \cdot x + 7$

2. Find the absolute minimum and maximum values and all locations where they occur. Explain if either does not exist.

Domain: $\mathbb{R}$ (all real x)

$f(x) = x^3 + 3 \cdot x^2 - 2$

3. Find the absolute minimum and maximum values and all locations where they occur. Explain if either does not exist.

Domain: $\mathbb{R}$ (all real x)

$f(x) = 2 \cdot x^4 + 8 \cdot x^3 - 16 \cdot x - 13$

4. Find the absolute minimum and maximum values and all locations where they occur. Explain if either does not exist.

Domain: $[-2, 2]$

$f(x) = x^2 - 2 \cdot x + 5$

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Absolute Extrema

Find the absolute minimum and maximum values on the stated domain, including every input where each value is attained. Explain when an extremum does not exist.

5. Find the absolute minimum and maximum values and all locations where they occur. Explain if either does not exist.

Domain: $[-6, 6]$

$$f(x) = -x^3 + 3x^2 - x - 5$$

6. Find the absolute minimum and maximum values and all locations where they occur. Explain if either does not exist.

Domain: $[-4, 4]$

$$f(x) = -2x^4 + 8x^2 - 1$$

7. Find the absolute minimum and maximum values and all locations where they occur. Explain if either does not exist.

Domain: $\mathbb{R}$ (all real x)

$$f(x) = -x^2 + 2$$

8. Find the absolute minimum and maximum values and all locations where they occur. Explain if either does not exist.

Domain: $\mathbb{R}$ (all real x)

$$f(x) = -x^3 + 2$$

Absolute Extrema
Worked Answer Key

Find the absolute minimum and maximum values on the stated domain, including every input where each value is attained. Explain when an extremum does not exist.

1. Domain: $\mathbb{R}$ (all real x)

$f(x) = x^2 + 4 \cdot x + 7$

$f'(x) = 2 \cdot x + 4$

Stationary x : -2

End behavior: left $\rightarrow +\infty$; right $\rightarrow +\infty$.

Candidate comparison:

$x = -2$, $f(x) = 3$ (stationary)

Absolute minimum: value 3 at $x = -2$

Absolute maximum: does not exist; f is unbounded above.

2. Domain: $\mathbb{R}$ (all real x)

$f(x) = x^3 + 3 \cdot x^2 - 2$

$f'(x) = 3 \cdot x^2 + 6 \cdot x$

Stationary x : -2 , 0

End behavior: left $\rightarrow -\infty$; right $\rightarrow +\infty$.

Candidate comparison:

$x = -2$, $f(x) = 2$ (stationary)

$x = 0$, $f(x) = -2$ (stationary)

Absolute minimum: does not exist; f is unbounded below.

Absolute maximum: does not exist; f is unbounded above.

3. Domain: $\mathbb{R}$ (all real x)

$f(x) = 2 \cdot x^4 + 8 \cdot x^3 - 16 \cdot x - 13$

$f'(x) = 8 \cdot x^3 + 24 \cdot x^2 - 16$

Stationary x : $-\sqrt{3} - 1$, -1 , $\sqrt{3} - 1$

End behavior: left $\rightarrow +\infty$; right $\rightarrow +\infty$.

Candidate comparison:

$x = -\sqrt{3} - 1$, $f(x) = -21$ (stationary)

$x = -1$, $f(x) = -3$ (stationary)

$x = \sqrt{3} - 1$, $f(x) = -21$ (stationary)

Absolute minimum: value -21 at $x = -\sqrt{3} - 1$, $\sqrt{3} - 1$

Absolute maximum: does not exist; f is unbounded above.

4. Domain: $[-2, 2]$

$f(x) = x^2 - 2 \cdot x + 5$

$f'(x) = 2 \cdot x - 2$

Stationary x : 1

Candidate comparison:

$x = -2$, $f(x) = 13$ (left endpoint)

$x = 1$, $f(x) = 4$ (stationary)

$x = 2$, $f(x) = 5$ (right endpoint)

Absolute minimum: value 4 at $x = 1$

Absolute maximum: value 13 at $x = -2$

Absolute Extrema
Worked Answer Key

Find the absolute minimum and maximum values on the stated domain, including every input where each value is attained. Explain when an extremum does not exist.

5. Domain: [-6, 6]

$f(x) = -x^3 + 3x^2 - x - 5$

$f'(x) = -3x^2 + 6x - 1$

Stationary x: $-\frac{1}{3}\sqrt{6} + 1, \frac{1}{3}\sqrt{6} + 1$

Candidate comparison:

$x = -6, f(x) = 325$ (left endpoint)

$x = -\frac{1}{3}\sqrt{6} + 1, f(x) = -\frac{4}{9}\sqrt{6} - 4$ (stationary)

$x = \frac{1}{3}\sqrt{6} + 1, f(x) = \frac{4}{9}\sqrt{6} - 4$ (stationary)

$x = 6, f(x) = -119$ (right endpoint)

Absolute minimum: value -119 at $x = 6$

Absolute maximum: value 325 at $x = -6$

6. Domain: [-4, 4]

$f(x) = -2x^4 + 8x^2 - 1$

$f'(x) = -8x^3 + 16x$

Stationary x: $-\sqrt{2}, 0, \sqrt{2}$

Candidate comparison:

$x = -4, f(x) = -385$ (left endpoint)

$x = -\sqrt{2}, f(x) = 7$ (stationary)

$x = 0, f(x) = -1$ (stationary)

$x = \sqrt{2}, f(x) = 7$ (stationary)

$x = 4, f(x) = -385$ (right endpoint)

Absolute minimum: value -385 at $x = -4, 4$

Absolute maximum: value 7 at $x = -\sqrt{2}, \sqrt{2}$

7. Domain: R (all real x)

$f(x) = -x^2 + 2$

$f'(x) = -2x$

Stationary x: 0

End behavior: left $\rightarrow$ -infinity; right $\rightarrow$ -infinity.

Candidate comparison:

$x = 0, f(x) = 2$ (stationary)

Absolute minimum: does not exist; f is unbounded below.

Absolute maximum: value 2 at $x = 0$

8. Domain: R (all real x)

$f(x) = -x^3 + 2$

$f'(x) = -3x^2$

Stationary x: 0

End behavior: left $\rightarrow$ +infinity; right $\rightarrow$ -infinity.

Candidate comparison:

$x = 0, f(x) = 2$ (stationary)

Absolute minimum: does not exist; f is unbounded below.

Absolute maximum: does not exist; f is unbounded above.