

Taylor and Maclaurin Polynomials

Build each requested polynomial from exact derivative coefficients. Show the center and keep factorial coefficients exact.

1. Find the Taylor polynomial through order 5 for  $f(x)=\ln(x)$  centered at  $a = 1$ .

2. Find the Taylor polynomial through order 5 for  $f(x)=\sin(x)$  centered at  $a = 0$ .

Taylor and Maclaurin Polynomials

Build each requested polynomial from exact derivative coefficients. Show the center and keep factorial coefficients exact.

3. Find the Taylor polynomial through order 5 for  $f(x)=1/x$  centered at  $a = 1$ .

4. Find the Taylor polynomial through order 5 for  $f(x)=e^x$  centered at  $a = 0$ .

Taylor and Maclaurin Polynomials

Build each requested polynomial from exact derivative coefficients. Show the center and keep factorial coefficients exact.

5. Find the Taylor polynomial through order 4 for  $f(x)=\cos(x)$  centered at  $a = 0$ .

6. Find the Taylor polynomial through order 4 for  $f(x)=e^x$  centered at  $a = 1$ .

## Taylor and Maclaurin Polynomials

### Worked Answer Key

Build each requested polynomial from exact derivative coefficients. Show the center and keep factorial coefficients exact.

1. Find the Taylor polynomial through order 5 for  $f(x)=\ln(x)$  centered at  $a = 1$ .

$$P_5(x) = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4 + \frac{1}{5}(x - 1)^5.$$

1. Use  $P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!}(x-a)^k$ .
2. Exact nonzero coefficients  $f^{(k)}(a)/k!$ :  $a_1=1$ ,  $a_2=-1/2$ ,  $a_3=1/3$ ,  $a_4=-1/4$ ,  $a_5=1/5$ .
3. Substitute  $a = 1$  and retain all terms through order 5; a zero coefficient may leave the displayed degree below 5.

## Taylor and Maclaurin Polynomials

### Worked Answer Key

Build each requested polynomial from exact derivative coefficients. Show the center and keep factorial coefficients exact.

2. Find the Taylor polynomial through order 5 for  $f(x)=\sin(x)$  centered at  $a = 0$ .

$$P_5(x) = x - (1/6)x^3 + (1/120)x^5.$$

1. Use  $P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k$ .
2. Exact nonzero coefficients  $f^{(k)}(a)/k!$ :  $a_1=1$ ,  $a_3=-1/6$ ,  $a_5=1/120$ .
3. Substitute  $a = 0$  and retain all terms through order 5; a zero coefficient may leave the displayed degree below 5.

## Taylor and Maclaurin Polynomials

### Worked Answer Key

Build each requested polynomial from exact derivative coefficients. Show the center and keep factorial coefficients exact.

3. Find the Taylor polynomial through order 5 for  $f(x)=1/x$  centered at  $a = 1$ .

$$P_5(x) = 1 - (x - 1) + (x - 1)^2 - (x - 1)^3 + (x - 1)^4 - (x - 1)^5.$$

1. Use  $P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k$ .
2. Exact nonzero coefficients  $f^{(k)}(a)/k!$ :  $a_0=1$ ,  $a_1=-1$ ,  $a_2=1$ ,  $a_3=-1$ ,  $a_4=1$ ,  $a_5=-1$ .
3. Substitute  $a = 1$  and retain all terms through order 5; a zero coefficient may leave the displayed degree below 5.

## Taylor and Maclaurin Polynomials

### Worked Answer Key

Build each requested polynomial from exact derivative coefficients. Show the center and keep factorial coefficients exact.

4. Find the Taylor polynomial through order 5 for  $f(x)=e^x$  centered at  $a = 0$ .

$$P_5(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5.$$

1. Use  $P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!}(x-a)^k$ .
2. Exact nonzero coefficients  $f^{(k)}(a)/k!$ :  $a_0=1$ ,  $a_1=1$ ,  $a_2=1/2$ ,  $a_3=1/6$ ,  $a_4=1/24$ ,  $a_5=1/120$ .
3. Substitute  $a = 0$  and retain all terms through order 5; a zero coefficient may leave the displayed degree below 5.

## Taylor and Maclaurin Polynomials

### Worked Answer Key

Build each requested polynomial from exact derivative coefficients. Show the center and keep factorial coefficients exact.

5. Find the Taylor polynomial through order 4 for  $f(x)=\cos(x)$  centered at  $a = 0$ .

$$P_4(x) = 1 - (1/2)x^2 + (1/24)x^4.$$

1. Use  $P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!}(x-a)^k$ .
2. Exact nonzero coefficients  $f^{(k)}(a)/k!$ :  $a_0=1$ ,  $a_2=-1/2$ ,  $a_4=1/24$ .
3. Substitute  $a = 0$  and retain all terms through order 4; a zero coefficient may leave the displayed degree below 4.

## Taylor and Maclaurin Polynomials

### Worked Answer Key

Build each requested polynomial from exact derivative coefficients. Show the center and keep factorial coefficients exact.

6. Find the Taylor polynomial through order 4 for  $f(x)=e^x$  centered at  $a = 1$ .

$$P_4(x) = e + (e)(x - 1) + (e/2)(x - 1)^2 + (e/6)(x - 1)^3 + (e/24)(x - 1)^4.$$

1. Use  $P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!}(x-a)^k$ .
2. Exact nonzero coefficients  $f^{(k)}(a)/k!$ :  $a_0=e$ ,  $a_1=e$ ,  $a_2=e/2$ ,  $a_3=e/6$ ,  $a_4=e/24$ .
3. Substitute  $a = 1$  and retain all terms through order 4; a zero coefficient may leave the displayed degree below 4.