

Name \_\_\_\_\_ Score \_\_\_\_\_ Teacher \_\_\_\_\_ Date \_\_\_\_\_

## 2 by 2 Eigenvalues and Eigenvectors

Find the characteristic polynomial, exact eigenvalues, and a legitimate eigenvector basis for every eigenspace.

1. For  $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

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2. For  $A = \begin{bmatrix} -3 & 0 \\ 12 & 3 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

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## 2 by 2 Eigenvalues and Eigenvectors

Find the characteristic polynomial, exact eigenvalues, and a legitimate eigenvector basis for every eigenspace.

3. For  $A = \begin{bmatrix} 2 & 8 \\ 0 & -2 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

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4. For  $A = \begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

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## 2 by 2 Eigenvalues and Eigenvectors

Find the characteristic polynomial, exact eigenvalues, and a legitimate eigenvector basis for every eigenspace.

5. For  $A = \begin{bmatrix} -6 & 4 \\ -1 & -2 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

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6. For  $A = \begin{bmatrix} 5 & 5 \\ 0 & 0 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

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## 2 by 2 Eigenvalues and Eigenvectors

### Worked Answer Key

Find the characteristic polynomial, exact eigenvalues, and a legitimate eigenvector basis for every eigenspace.

1. For  $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

$p(\lambda) = \lambda^2 + 2\lambda + 1 = (\lambda + 1)(\lambda + 1)$ ;  $\lambda = -1$  (algebraic multiplicity 2),  $E_{-1} = \text{span}\{(1, 0), (0, 1)\}$ . Every nonzero vector in  $\mathbb{R}^2$  is an eigenvector; the zero vector is not an eigenvector.

1.  $\det(A - \lambda I) = (-1 - \lambda)(-1 - \lambda) - (0)(0) = \lambda^2 + 2\lambda + 1 = (\lambda + 1)(\lambda + 1)$ .

2.  $A - -1I$  is the zero matrix, so  $E_{-1} = \mathbb{R}^2$  has basis  $\{(1, 0), (0, 1)\}$ .

3. Check:  $A(1, 0) = (-1, 0) = -1(1, 0)$ .

4. Check:  $A(0, 1) = (0, -1) = -1(0, 1)$ .

5. Every nonzero vector is an eigenvector; zero is excluded by definition.

## 2 by 2 Eigenvalues and Eigenvectors

### Worked Answer Key

Find the characteristic polynomial, exact eigenvalues, and a legitimate eigenvector basis for every eigenspace.

2. For  $A = \begin{bmatrix} -3 & 0 \\ 12 & 3 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

$p(\lambda) = \lambda^2 - 9 = (\lambda - 3)(\lambda + 3)$ ;  $\lambda = 3$  (algebraic multiplicity 1),  $E_3 = \text{span}\{(0, -1)\}$ ;  $\lambda = -3$  (algebraic multiplicity 1),  $E_{-3} = \text{span}\{(-1, 2)\}$ . For  $\lambda = 3$ , all eigenvectors are the nonzero multiples of  $(0, -1)$ . For  $\lambda = -3$ , all eigenvectors are the nonzero multiples of  $(-1, 2)$ . The zero vector is not an eigenvector.

1.  $\det(A - \lambda I) = (-3 - \lambda)(3 - \lambda) - (0)(12) = \lambda^2 - 9 = (\lambda - 3)(\lambda + 3)$ .
2. Solving each  $(A - \lambda I)v = 0$  gives  $E_3 = \text{span}\{(0, -1)\}$  and  $E_{-3} = \text{span}\{(-1, 2)\}$ .
3. Check:  $A(0, -1) = (0, -3) = 3(0, -1)$ .
4. Check:  $A(-1, 2) = (3, -6) = -3(-1, 2)$ .
5. Each eigenvalue has algebraic and geometric multiplicity 1.

## 2 by 2 Eigenvalues and Eigenvectors

### Worked Answer Key

Find the characteristic polynomial, exact eigenvalues, and a legitimate eigenvector basis for every eigenspace.

3. For  $A = \begin{bmatrix} 2 & 8 \\ 0 & -2 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

$p(\lambda) = \lambda^2 - 4 = (\lambda + 2)(\lambda - 2)$ ;  $\lambda = -2$  (algebraic multiplicity 1),  $E_{-2} = \text{span}\{(-2, 1)\}$ ;  $\lambda = 2$  (algebraic multiplicity 1),  $E_2 = \text{span}\{(1, 0)\}$ . For  $\lambda = -2$ , all eigenvectors are the nonzero multiples of  $(-2, 1)$ . For  $\lambda = 2$ , all eigenvectors are the nonzero multiples of  $(1, 0)$ . The zero vector is not an eigenvector.

1.  $\det(A - \lambda I) = (2 - \lambda)(-2 - \lambda) - (8)(0) = \lambda^2 - 4 = (\lambda + 2)(\lambda - 2)$ .
2. Solving each  $(A - \lambda I)v = 0$  gives  $E_{-2} = \text{span}\{(-2, 1)\}$  and  $E_2 = \text{span}\{(1, 0)\}$ .
3. Check:  $A(-2, 1) = (4, -2) = -2(-2, 1)$ .
4. Check:  $A(1, 0) = (2, 0) = 2(1, 0)$ .
5. Each eigenvalue has algebraic and geometric multiplicity 1.

## 2 by 2 Eigenvalues and Eigenvectors

### Worked Answer Key

Find the characteristic polynomial, exact eigenvalues, and a legitimate eigenvector basis for every eigenspace.

4. For  $A = \begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

$p(\lambda) = \lambda^2 - 1 = (\lambda - 1)(\lambda + 1)$ ;  $\lambda = 1$  (algebraic multiplicity 1),  $E_1 = \text{span}\{(-1, -1)\}$ ;  $\lambda = -1$  (algebraic multiplicity 1),  $E_{-1} = \text{span}\{(1, 2)\}$ . For  $\lambda = 1$ , all eigenvectors are the nonzero multiples of  $(-1, -1)$ . For  $\lambda = -1$ , all eigenvectors are the nonzero multiples of  $(1, 2)$ . The zero vector is not an eigenvector.

1.  $\det(A - \lambda I) = (3 - \lambda)(-3 - \lambda) - (-2)(4) = \lambda^2 - 1 = (\lambda - 1)(\lambda + 1)$ .
2. Solving each  $(A - \lambda I)v = 0$  gives  $E_1 = \text{span}\{(-1, -1)\}$  and  $E_{-1} = \text{span}\{(1, 2)\}$ .
3. Check:  $A(-1, -1) = (-1, -1) = 1(-1, -1)$ .
4. Check:  $A(1, 2) = (-1, -2) = -1(1, 2)$ .
5. Each eigenvalue has algebraic and geometric multiplicity 1.

## 2 by 2 Eigenvalues and Eigenvectors

### Worked Answer Key

Find the characteristic polynomial, exact eigenvalues, and a legitimate eigenvector basis for every eigenspace.

5. For  $A = \begin{bmatrix} -6 & 4 \\ -1 & -2 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

$p(\lambda) = \lambda^2 + 8\lambda + 16 = (\lambda + 4)(\lambda + 4)$ ;  $\lambda = -4$  (algebraic multiplicity 2),  $E_{-4} = \text{span}\{(2, 1)\}$ . For  $\lambda = -4$ , all eigenvectors are the nonzero multiples of  $(2, 1)$ . The zero vector is not an eigenvector.

1.  $\det(A - \lambda I) = (-6 - \lambda)(-2 - \lambda) - (4)(-1) = \lambda^2 + 8\lambda + 16 = (\lambda + 4)(\lambda + 4)$ .

2. Solving each  $(A - \lambda I)v = 0$  gives  $E_{-4} = \text{span}\{(2, 1)\}$ .

3. Check:  $A(2, 1) = (-8, -4) = -4(2, 1)$ .

4. The repeated eigenvalue has a one-dimensional eigenspace, so the matrix is defective.

## 2 by 2 Eigenvalues and Eigenvectors

### Worked Answer Key

Find the characteristic polynomial, exact eigenvalues, and a legitimate eigenvector basis for every eigenspace.

6. For  $A = \begin{bmatrix} 5 & 5 \\ 0 & 0 \end{bmatrix}$ , find  $p(\lambda)$ , the exact eigenvalues, and a basis for each eigenspace.

$p(\lambda) = \lambda^2 - 5\lambda = (\lambda - 0)(\lambda - 5)$ ;  $\lambda = 0$  (algebraic multiplicity 1),  $E_0 = \text{span}\{(1, -1)\}$ ;  $\lambda = 5$  (algebraic multiplicity 1),  $E_5 = \text{span}\{(-1, 0)\}$ . For  $\lambda = 0$ , all eigenvectors are the nonzero multiples of  $(1, -1)$ . For  $\lambda = 5$ , all eigenvectors are the nonzero multiples of  $(-1, 0)$ . The zero vector is not an eigenvector.

1.  $\det(A - \lambda I) = (5 - \lambda)(0 - \lambda) - (5)(0) = \lambda^2 - 5\lambda = (\lambda - 0)(\lambda - 5)$ .
2. Solving each  $(A - \lambda I)v = 0$  gives  $E_0 = \text{span}\{(1, -1)\}$  and  $E_5 = \text{span}\{(-1, 0)\}$ .
3. Check:  $A(1, -1) = (0, 0) = 0(1, -1)$ .
4. Check:  $A(-1, 0) = (-5, 0) = 5(-1, 0)$ .
5. Each eigenvalue has algebraic and geometric multiplicity 1.